Modèle VHP: hydrodynamique

$f^{(0)}(\underline{v}) = \frac{n}{v_T^d}\frac{1}{\pi^{d/2}} e^{-v^2/v_T^2}$; $\mathcal{M}(v/v_T) = \frac{1}{\pi^{d/2}} e^{-v^2/v_T^2}\frac{n}{v_T^d}$; $v_T=\sqrt{2/\beta m}$; $f^{(0)}(v)=\mathcal{M}(v/v_T)$

$$(\partial_t + \underline{v}\cdot\underline{\nabla}) f(\underline{r},\underline{v};t) = p J_a[f,f] + (1-p) J_c[f,f]$$

$$J_a[f,g] = -\sigma^{d-1}\frac{\varnothing}{S_d v_T}\int_{\mathbb{R}^d} d\underline{v}_2 \int d\hat{\sigma}\, |v_{12}|^2 f(\underline{r},\underline{v}_2;t)\, g(\underline{r},\underline{v}_1;t) = -\sigma^{d-1}\frac{\varnothing}{v_T} g(\underline{r},\underline{v}_1;t)\int_{\mathbb{R}^d} d\underline{v}_2\, v_{12}^2 f(\underline{r},\underline{v}_2;t)$$

$$J_c[f,g] = \sigma^{d-1}\frac{\varnothing}{S_d v_T}\int_{\mathbb{R}^d} dv_2 \int d\hat\sigma\, |v_{12}|^2 (b^{-1}-1)\, g(\underline{r},\underline{v}_1;t) f(\underline{r},\underline{v}_2;t)$$; $S_d = 2\pi^{d/2}/\Gamma(d/2)$

$$\omega[f,g] = -\int_{\mathbb{R}^d} dv_1\, J_a[f,g] = \sigma^{d-1}\frac{\varnothing}{v_T}\int_{\mathbb{R}^{2d}} dv_1 dv_2\, v_{12}^2\, g(\underline{r},\underline{v}_1;t) f(\underline{r},\underline{v}_2;t)$$

Solution Chapman-Enskog: ordre zéro: même forme:

$\partial_t n = -p\,\xi_n^{(0)}$,

$\partial_t u_i = -p\, v_T\, \xi_{u_i}^{(0)}$,

$\partial_t T = -p\, T\,\xi_T^{(0)}$,

avec:

$$\xi_n^{(0)} = \frac1n\, \omega[f^{(0)},f^{(0)}] = \frac1n\sigma^{d-1}\frac{\varnothing}{v_T}\int_{\mathbb{R}^{2d}} dv_1dv_2\, (v_1^2+v_2^2 - v_1\cdot v_2\cdot 2)\, f^{(0)}(\underline{v}_2;t)\, f^{(0)}(\underline{v}_1;t)$$

$$= \frac{1}{\cancel{n}}\sigma^{d-1}\frac{\varnothing}{\cancel{v_T}}\frac{n\cancel{n}}{\cancel{v_T^{2d}}}\frac{1}{\pi^d}\cancel{v_T^{2d}}\, v_T^{\cancel{2}} \int_{\mathbb{R}^{2d}} dc_1dc_2\, (c_1^2+c_2^2-2\cancelto{0}{c_1\cdot c_2})\, e^{-c_1^2}e^{-c_2^2}$$

$$= \sigma^{d-1}\varnothing\, n\, v_T \frac{1}{\pi^d}\left[\int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}c_1^2 + \int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}c_2^2\right]$$

$$= \sigma^{d-1}\varnothing\, n\, v_T \frac{1}{\cancel{\pi^d}}\cancel{\pi^{d/2}}\left[\cancel{\pi^{d/2}}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\cdot 2\right]$$

$$= \sigma^{d-1}\varnothing\, n\, v_T\, \frac d2 \cdot 2$$

$$= n\,\sigma^{d-1}\varnothing\, v_T\, d$$

$$\xi_{u_i}^{(0)} = \frac{1}{n v_T}\,\omega[f^{(0)}, v_i f^{(0)}] = \frac{1}{n v_T}\sigma^{d-1}\frac{\varnothing}{v_T}\int_{\mathbb{R}^{2d}} dv_1dv_2\, (v_1^2+v_2^2-2v_1v_2)\, v_{1i}\frac{1}{\pi^d}\frac{n^2}{v_T^{2d}} e^{-v_1^2/v_T^2}e^{-v_2^2/v_T^2}$$

$$= \frac{1}{\cancel{n v_T}}\sigma^{d-1}\frac{\varnothing}{\cancel{v_T}}\frac{1}{\pi^d}\frac{n\cancel{n}}{\cancel{v_T^{2d}}}\cancel{v_T^{2d}}\, v_T^{\cancel{3}}\int_{\mathbb{R}^{2d}} dc_1dc_2\, (\cancelto{0}{c_1^2}+\cancelto{0}{c_2^2} - 2c_1\cdot c_2)\, c_{1i}\, e^{-c_1^2}e^{-c_2^2}$$

$$= \sigma^{d-1}\varnothing\frac{v_T}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}(-2)\, c_{1j}c_{2j}c_{1i}$$

$$= -2\sigma^{d-1}\varnothing\frac{v_T}{\pi^d}\int_{\mathbb{R}^d} dc_1\, c_{1j}c_{1i}\, e^{-c_1^2}\underbrace{\int_{\mathbb{R}^d} dc_2\, e^{-c_2^2}c_{2j}}_{=0}$$

$$= 0$$

$$\xi_T^{(0)} = \frac{m}{n k_B T d}\,\omega[f^{(0)}, v^2 f^{(0)}] - \xi_n^{(0)}$$

$$= \frac{m}{n k_B T d}\sigma^{d-1}\frac{\varnothing}{v_T}\int_{\mathbb{R}^{2d}} dv_1dv_2\, (v_1^2+v_2^2-2\cancelto{0}{v_1\cdot v_2})\, v_1^2\frac{1}{\pi^d}\frac{n^2}{v_T^{2d}}e^{-v_1^2/v_T^2}e^{-v_2^2/v_T^2} - \xi_n^{(0)}$$

$$= \frac{m}{\cancel{n} k_B T d}\sigma^{d-1}\frac{\varnothing}{\cancel{v_T}}\frac{1}{\pi^d}\frac{n\cancel{n}}{\cancel{v_T^{2d}}}\cancel{v_T^{2d}}\, v_T^{3}\int_{\mathbb{R}^{2d}} dc_1dc_2\, [c_1^4+c_2^2c_1^2]\, e^{-c_1^2}e^{-c_2^2} - \xi_n^{(0)}$$

$$= \frac{m}{k_B T d}\sigma^{d-1}\frac{\varnothing}{\pi^d} n v_T^3 \left[\int_{\mathbb{R}^d} dc_1\, e^{-c_1^2}c_1^4\int_{\mathbb{R}^d} dc_2\, e^{-c_2^2} + \int_{\mathbb{R}^d} dc_1\, e^{-c_1^2}c_1^2\int_{\mathbb{R}^d} dc_2\, e^{-c_2^2}c_2^2\right] - \xi_n^{(0)}$$

$$= \frac{m}{k_B T d}\sigma^{d-1}\frac{\varnothing}{\pi^d} n v_T^3\left[\pi^d\frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)} + \pi^d\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\right] - \xi_n^{(0)}$$

$$= \frac{m}{k_B T d}\sigma^{d-1}\frac{\varnothing}{\cancel{\pi^d}}\, n v_T^3\, \cancel{\pi^d}\left[\frac{d+2}{2}\frac d2 + \frac d2\frac d2\right] - \xi_n^{(0)}$$

$$= \overbrace{\frac{m\beta}{2}}^{=1/v_T^2} \cancel{\frac{2}{d}} \sigma^{d-1} \phi\, n v_T^3 \cancel{\frac{d}{2}} \left(\frac{d+2}{2} + \frac{d}{2} \right) - \zeta_n^{(0)}$$

$$= n\sigma^{d-1} \phi\, v_T \, \frac{2d+2}{2} - n\sigma^{d-1}\phi\, v_T\, d$$

$$= n\sigma^{d-1}\phi\, v_T \left[d+1-d \right]$$

$$= n\sigma^{d-1}\phi\, v_T$$

Cette fois la température n'est plus conservée, mais l'état homogène est quand-même maxwellien. La solution de l'état homogène est

$$n_H(t) = n_0 \left(1 + p\, t/t_0\right)^{-\gamma_n} , \quad \gamma_n = \zeta_n^{(0)}(0)\, t_0 \quad , \; t_0^{-1} = \zeta_n^{(0)}(0) + \zeta_T^{(0)}(0)/2$$

$$T_H(t) = T_0 \left(1 + p\, t/t_0\right)^{-\gamma_T} , \quad \gamma_T = \zeta_T^{(0)}(0)\, t_0$$

Ordre 1 : les calculs sont similaires (cf. Maxwell) :

$$\eta^* = \frac{\eta}{\eta_0} = \frac{1}{\nu_\eta^* - \frac{1}{2} p\, \zeta_T^{(0)*}}$$

$$\kappa^* = \frac{\kappa}{\kappa_0} = \frac{1}{\nu_\kappa^* - 2p\,\zeta_T^{(0)*}} \left[\frac{1}{2} p\, \zeta_n^{(0)*} \mu^* + \frac{d-1}{d} (2\overset{=0}{\cancel{a_2}} + 1) \right] = \frac{1}{\nu_\kappa^* - 2p\,\zeta_T^{(0)*}} \left[\frac{1}{2} p\, \zeta_n^{(0)*} \mu^* + \frac{d-1}{d} \right]$$

$$\mu^* = \frac{n\mu}{T\kappa_0} = \frac{2}{2\nu_\mu^* - 3p\,\zeta_T^{(0)*} - 2p\,\zeta_n^{(0)*}} \left[p\, \zeta_T^{(0)*} \kappa^* + \frac{d-1}{d} \overset{=0}{\cancel{a_2}} \right] = \frac{2}{2\nu_\mu^* - 3p\,\zeta_T^{(0)*} - 2p\,\zeta_n^{(0)*}}\, p\, \zeta_T^{(0)*} \kappa^*$$

Calcul des coefficients :

$$\nu_\eta^* = \frac{1}{\nu_0} \frac{\int_{\mathbb{R}^d} dv\, D_{ij}(v)\, J\, 2_{ij}}{\int_{\mathbb{R}^d} dv\, D_{ij}(v)\, 2_{ij}} - p\, \frac{1}{\nu_0} \frac{\int_{\mathbb{R}^d} dv\, D_{ij}(v)\, \Omega\, 2_{ij}}{\int_{\mathbb{R}^d} dv\, D_{ij}(v)\, 2_{ij}} = (1-p)\, \nu_\eta^{*c} + p\, \nu_\eta^{*a}$$

$$\begin{cases} \nu_\eta^{*a} = \dfrac{\beta^2}{(d+2)(d-1) n \nu_0} \displaystyle\int_{\mathbb{R}^d} d\underline{v}\; D_{ij}(\underline{v})\, L_a[\mathcal{M} D_{ij}] + \nu_\eta^{*a'} \\ \nu_\eta^{*a'} = -\dfrac{\beta^2}{(d+2)(d-1) n \nu_0} \displaystyle\int_{\mathbb{R}^d} d\underline{v}\; D_{ij}(\underline{v})\, \Omega[\mathcal{M} D_{ij}] \\ \nu_\eta^{*c} = \dfrac{\beta^2}{(d+2)(d-1) n \nu_0} \displaystyle\int_{\mathbb{R}^d} d\underline{v}\; D_{ij}(\underline{v})\, L_c[\mathcal{M} D_{ij}] \end{cases}$$

$$, \quad D_{ij}(v) = m\left(v_i v_j - \frac{1}{d}\delta_{ij} v^2\right)$$

$$\nu_0^{-1} = \frac{t_0}{p^{(0)}} = \frac{1}{n k_B T} \frac{d+2}{8} \frac{\Gamma(d/2)}{\pi^{(d-1)/2}} \frac{\sqrt{m k_B T}}{\sigma^{d-1}}$$

$$\nu_\kappa^* = \frac{1}{\nu_0} \frac{\int_{\mathbb{R}^d} dv\, S_i(v)\, J\, \mathcal{A}_i}{\int_{\mathbb{R}^d} d\underline{v}\, S_i(v)\, \mathcal{A}_i} - p\, \frac{1}{\nu_0} \frac{\int_{\mathbb{R}^d} dv\, S_i(\underline{v})\, \Omega\, \mathcal{A}_i}{\int_{\mathbb{R}^d} d\underline{v}\, S_i(\underline{v})\, \mathcal{A}_i} = (1-p)\, \nu_\kappa^{*c} + p\, \nu_\kappa^{*a}$$

$$\begin{cases} \nu_\kappa^{*a} = \dfrac{2m\beta^3}{d(d+2)\, n \nu_0} \displaystyle\int_{\mathbb{R}^d} d\underline{v}\; S_i(\underline{v})\, L_a[\mathcal{M} S_i] + \nu_\kappa^{*a'} \\ \nu_\kappa^{*a'} = -\dfrac{2m\beta^3}{d(d+2)\, n \nu_0} \displaystyle\int_{\mathbb{R}^d} d\underline{v}\; S_i(\underline{v})\, \Omega[\mathcal{M} S_i] \\ \nu_\kappa^{*c} = \dfrac{2m\beta^3}{d(d+2)\, n \nu_0} \displaystyle\int_{\mathbb{R}^d} d\underline{v}\; S_i(\underline{v})\, L_c[\mathcal{M} S_i] \end{cases}$$

$$, \quad S_i(v) = \left(\frac{m}{2} v^2 - \frac{d+2}{2} k_B T\right) v_i = \frac{m}{2}\left(v^2 - \frac{d+2}{2} v_T^2\right) v_i$$

Lemme 3.3 :
$$\int_{\mathbb{R}^d} d\underline{v}\; Y(\underline{v}_1)\, L_a[\mathcal{M}X] = \sigma^{d-1} \frac{\phi}{v_T} \int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\; |v_{12}|^2 f^{(0)}(\underline{v}_1)\, \mathcal{M}(\underline{v}_2)\, X(\underline{v}_2) \left[Y(v_1) + Y(v_2) \right]$$

Lemme 3.4 :
$$\int_{\mathbb{R}^d} d\underline{v}\; Y(\underline{v}_1)\, L_c[\mathcal{M}X] = -\sigma^{d-1} \frac{\phi}{v_T S_d} \int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\; f^{(0)}(\underline{v}_1)\, \mathcal{M}(\underline{v}_2) X(\underline{v}_2) \int d\hat{\sigma}\; |v_{12}|^2 (b-1) \left[Y(\underline{v}_1) + Y(\underline{v}_2) \right] , \quad b\underline{v}_i = \underline{v}_i \mp (\underline{v}_{12} \cdot \hat{\sigma})\hat{\sigma}$$

Calcul de V_2^{*a} :

$$\int_{\mathbb{R}^d} dv\, D_{ij}(\underline{V}) L_a[MD_{ij}] \overset{3.3.}{=} \sigma^{d-1}\frac{\varnothing}{V_T}\int_{\mathbb{R}^{2d}} d\underline{v_1}d\underline{v_2}\, |V_{12}|^2 f^{(0)}(V_1)M(V_2)D_{ij}(V_2)\left[D_{ij}(V_1)+D_{ij}(V_2)\right]$$

$$= \sigma^{d-1}\frac{\varnothing}{V_T}\frac{n^2}{V_T^{2d}}\frac{1}{\pi^d}\int_{\mathbb{R}^{2d}} d\underline{v_1}d\underline{v_2}\,(V_1^2+V_2^2-2V_1\cdot V_2)e^{-V_1^2/V_T^2}e^{-V_2^2/V_T^2} m^2\left[V_{2i}V_{2j}-\frac{1}{d}\delta_{ij}V_2^2\right]\left[V_{1i}V_{1j}-\frac{1}{d}\delta_{ij}V_1^2+V_{2i}V_{2j}-\frac{1}{d}\delta_{ij}V_2^2\right]$$

$$= \sigma^{d-1}\frac{\varnothing}{\cancel{V_T}}\frac{n^2}{\cancel{V_T^{2d}}}\frac{1}{\pi^d}\cancel{V_T^{2d}}\, m V_T^{2} V_T^4\int_{\mathbb{R}^{2d}} dc_1dc_2\,(c_1^2+c_2^2-2c_1\cdot c_2)e^{-c_1^2}e^{-c_2^2}\left[(V_1\cdot V_2)^2+V_2^4-\frac{1}{d}V_1^2V_2^2-\frac{1}{d}V_2^4-\frac{1}{d}V_1^2V_2^2-\frac{1}{d}V_2^4+\frac{1}{d^2}V_1^2V_2^2d+\frac{1}{d^2}V_2^4d\right]_{V=c}$$

$$= \sigma^{d-1}\varnothing n^2m^2\frac{V_T^5}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1dc_2\,(c_1^2+c_2^2-2c_1\cdot c_2)e^{-c_1^2}e^{-c_2^2}\left[(c_1\cdot c_2)^2+c_2^4\left(1-\frac{1}{d}-\frac{1}{d}+\frac{1}{d}\right)-c_1^2c_2^2\left(\frac{1}{d}+\frac{1}{d}-\frac{1}{d}\right)\right]$$

$$= \sigma^{d-1}\varnothing n^2m^2\frac{V_T^5}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\left[(c_1\cdot c_2)^2-c_1^2c_2^2\frac{1}{d}+\frac{d-1}{d}c_2^4\right]$$

$$= \sigma^{d-1}\varnothing n^2m^2\frac{V_T^5}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}\left[\frac{d-1}{d}c_1^2c_2^4+\frac{d-1}{d}c_2^6-\cancel{\frac{1}{d}c_1^4c_2^2}-\cancel{\frac{1}{d}c_1^2c_2^4}+\cancel{\frac{1}{d}c_1^4c_2^2}+\cancel{\frac{1}{d}c_1^2c_2^4}+\text{impair}\right]$$

$$= \sigma^{d-1}\varnothing n^2m^2\frac{V_T^5}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}\frac{d-1}{d}\left(c_1^2c_2^4+c_2^6\right)$$

$$= \sigma^{d-1}\varnothing n^2m^2\frac{V_T^5}{\cancel{\pi^d}}\frac{d-1}{d}\cancel{\pi^d}\left[\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)}+\frac{\Gamma(\frac{d+6}{2})}{\Gamma(d/2)}\right]$$

$$= \sigma^{d-1}\varnothing n^2m^2V_T^5\frac{d-1}{d}\left[\frac{d}{2}\frac{d+2}{2}\frac{d}{2}+\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}\right]$$

$$= \sigma^{d-1}\varnothing n^2m^2V_T^5\frac{d-1}{\cancel{d}}\frac{\cancel{d}}{2}\frac{d+2}{2}\underbrace{\left[\frac{d}{2}+\frac{d+4}{2}\right]}_{=\frac{2d+4}{2}=d+2}$$

$$= \sigma^{d-1}\varnothing n^2m^2V_T^5\frac{d-1}{2}\frac{(d+2)^2}{2} \quad ✓$$

$$\frac{\beta^2}{(d+2)(d-1)nV_0}\int_{\mathbb{R}^d} d\underline{v}\, D_{ij}(\underline{V})L_a[MD_{ij}] = \frac{\beta^2}{\cancel{(d+2)}\cancel{(d-1)}\cancel{n}V_0}\sigma^{d-1}\varnothing n^{\cancel{2}}m^2V_T^5\frac{\cancel{d-1}}{2}\frac{(d+2)^{\cancel{2}}}{2}$$

$$= \frac{\sigma^{d-1}\beta^2\varnothing n m^2V_T^5}{V_0}\frac{d+2}{4}$$

$$= \cancel{\sigma^{d-1}}\beta^2\varnothing\cancel{n}m^2V_T^5\frac{(d+2)}{4}\frac{1}{\cancel{n}k_BT}\frac{d+2}{8}\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\frac{\sqrt{mk_BT}}{\cancel{\sigma^{d-1}}} \quad , V_T=\sqrt{2/\beta m}$$

$$= \cancel{\beta^2}\varnothing\cancel{m^2}\frac{\cancel{X}}{\cancel{\beta^2}\cancel{m^2}}\sqrt{\frac{2}{\cancel{\beta m}}}\frac{(d+2)^2}{\cancel{4}8}\cancel{\beta}\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\sqrt{\frac{\cancel{m}}{\cancel{\beta}}}$$

$$= \varnothing\frac{(d+2)^2}{8}\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} \quad ✓$$

Calcul de V_2^{*c} : le calcul est en tout point similaire à ce qui a été fait pour le modèle de Maxwell, à la différence de la vitesse relative V_{12}^2. On peut donc réutiliser un résultat intermédiaire (l'intégrale angulaire) :

$$\int_{\mathbb{R}^d} d\underline{v}\, D_{ij}(\underline{V})L_c[MD_{ij}] = -\sigma^{d-1}\frac{\varnothing V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\overset{=2\pi^{(d-1)/2}\Gamma(\frac{n+1}{2})/\Gamma(\frac{n+d}{2})}{\beta_2}}{d+2}\int_{\mathbb{R}^{2d}} dc_1dc_2\, e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\left(c_{2i}c_{2j}-\frac{1}{d}c_2^2\delta_{ij}\right)$$
$$\times\Big(c_1^2\delta_{ij}+c_2^2\delta_{ij}-\overset{=c_1\cdot c_2}{2c_{1k}c_{2k}}\delta_{ij}-d\,c_{1i}c_{1j}+d\,c_{1i}c_{2j}+d\,c_{2i}c_{1j}-d\,c_{2i}c_{2j}\Big)$$

$$= -\sigma^{d-1}\frac{\varnothing V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}\int_{\mathbb{R}^{2d}} d\underline{c_1}d\underline{c_2}\, e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\Big[c_1^2c_2^2+c_2^4-\cancel{2(c_1\cdot c_2)c_2^2}-d(c_1\cdot c_2)^2+d(c_1\cdot c_2)c_2^2$$
$$+d(c_1\cdot c_2)c_2^2-d\,c_2^4-\cancel{\frac{1}{d}c_1^2c_2^2d}-\cancel{\frac{1}{d}c_2^4d}$$
$$+\cancel{\frac{2}{d}c_2^2(c_1\cdot c_2)d}+\cancel{c_2^2c_1^2}-c_2^2(c_1\cdot c_2)-c_2^2(c_1\cdot c_2)+\cancel{c_2^4}\Big]$$

$$= -\sigma^{d-1}\frac{\varnothing V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}\int_{\mathbb{R}^{2d}} d\underline{c_1}d\underline{c_2}\, e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\left[c_1^2c_2^2-(d-1)c_2^4-d(c_1\cdot c_2)^2+2(d-1)c_2^2(c_1\cdot c_2)\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}\int_{\mathbb{R}^{2d}} d\underline{c}_1 d\underline{c}_2\, e^{-c_1^2}e^{-c_2^2}\Big[(c_1^2+c_2^2)\big[-(d-1)c_2^4\big] - 2(c_1\cdot c_2)\,2\,(d-1)\,c_2^2(c_1\cdot c_2)\Big]$$

$$= -\sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}\int_{\mathbb{R}^{2d}} d\underline{c}_1 d\underline{c}_2\, e^{-c_1^2}e^{-c_2^2}\Big[-(d-1)c_1^2c_2^4 - (d-1)c_2^6 - 4(d-1)c_2^2(c_1\cdot c_2)^2\Big]$$

$$= \sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}(d-1)\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}\Big[c_1^2c_2^4 + c_2^6 + \frac{4}{d}c_2^4c_1^2\Big]$$

$$= \sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}(d-1)\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}\Big[c_2^6 + \frac{d+4}{d}c_1^2c_2^4\Big]$$

$$= \sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{d+2}(d-1)\Big[\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2} + \frac{d+4}{d}\frac{d}{2}\frac{d+2}{2}\frac{d}{2}\Big]\pi^d$$

$$= \sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n^2m^2}{\pi^d}\frac{2\beta_2}{\cancel{d+2}}(d-1)\frac{d+4}{2}\frac{\cancel{d+2}}{2}\frac{d}{2}\Big[1+\frac{d}{2}\Big]\pi^d \quad ✓$$

$$\frac{\beta^2}{(d+2)(d-1)nV_0}\int_{\mathbb{R}^d} d\underline{v}\, D_{ij}(\underline{v})\, L_c[\mathcal{M}D_{ij}] = \frac{\beta^2}{\cancel{(d+2)}\cancel{(d-1)}\cancel{n}V_0}\sigma^{d-1}\frac{\phi V_T^5}{S_d}\frac{n\cancel{n}m^2}{\cancel{\pi^d}}\cancel{2}\beta_2\cancel{(d-1)}\frac{1}{\cancel{4}}\frac{d+4}{2}\frac{d}{2}\frac{\cancel{d+2}}{2}\cancel{\pi^d}$$

↓ il y avait ici une erreur d'algèbre (corrigée ici)

$$= \cancel{\beta^2}\frac{1}{\cancel{nk_BT}}\frac{d+2}{\cancel{8}}\frac{\Gamma(d/2)}{\cancel{\pi^{(d-1)/2}}}\frac{\sqrt{\cancel{nk_BT}}}{\cancel{\sigma^{d-1}}}\cancel{\sigma^{d-1}}\phi\frac{\cancel{4}}{\cancel{\beta^2}\cancel{m^2}}\sqrt{\frac{2}{\beta m}}\frac{1}{2\pi^{d/2}}\Gamma\Big(\frac{d}{2}\Big)\cancel{n}\cancel{m^2}\cancel{2\pi^{(d-1)/2}}\frac{\Gamma\big(\frac{1+2}{2}\big)}{\Gamma\big(\frac{d+2}{2}\big)}\frac{d+4}{2}\frac{d}{2}\frac{1}{2}$$

$$= (d+2)\,\Gamma(d/2)\,\phi\,\sqrt{2}\,\frac{1}{2\pi^{d/2}}\cancel{\Gamma(d/2)}\;\frac{\cancel{1}}{\cancel{2}}\sqrt{\pi}\,\frac{1}{\cancel{\frac{d}{2}}\cancel{\Gamma(d/2)}}\frac{(d+4)}{8}\cancel{d}$$

$$= \phi\,\frac{(d+2)(d+4)}{16}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} \quad ✓$$

⚠ ici le ϕ peut être choisi différent (pour avoir l'unité $p=1$) (en fait, il doit être différent de Maxwell car c'est la fréq. collision, qui dépend du modèle)

Calcul de V_K^{vra} :

$$\int_{\mathbb{R}^d} d\underline{v}\, S_i(\underline{v})\, L_a[S_i] \overset{3.3.}{=} \sigma^{d-1}\frac{\phi}{V_T}\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\, |v_{12}|^2 f^{(m)}(\underline{v}_1)\,\mathcal{M}(\underline{v}_2)\,S_i(\underline{v}_2)\,[S_i(\underline{v}_1)+S_i(\underline{v}_2)]$$

$$= \sigma^{d-1}\frac{\phi}{V_T}\int_{\mathbb{R}^{2d}} dv_1 dv_2\,(V_1^2+V_2^2-2V_1\cdot V_2)\frac{n^2}{V_T^{d2}}\frac{1}{\pi^d}e^{-V_1^2/V_T^2}e^{-V_2^2/V_T^2}\frac{m^2}{4}\Big(V_2^2-\frac{d+2}{2}V_T^2\Big)V_{2i}\Big[\Big(V_1^2-\frac{d+2}{2}V_T^2\Big)V_{1i}+\Big(V_2^2-\frac{d+2}{2}V_T^2\Big)V_{2i}\Big]$$

$$= \sigma^{d-1}\frac{\phi}{V_T}\frac{n^2}{\cancel{V_T^{2d}}}\frac{1}{\pi^d}\cancel{V_T^{2d}}\,V_T^2V_T^2V_TV_T^2V_T\int_{\mathbb{R}^{2d}} dc_1dc_2\,(c_1^2+c_2^2-2c_1\cdot c_2)e^{-c_1^2}e^{-c_2^2}\frac{m^2}{4}\Big(c_2^2-\frac{d+2}{2}\Big)c_{2i}\Big[\Big(c_1^2-\frac{d+2}{2}\Big)c_{1i}+\Big(c_2^2-\frac{d+2}{2}\Big)c_{2i}\Big]$$

$$= \sigma^{d-1}\frac{\phi}{V_T}\frac{n^2}{\pi^d}V_T^8\frac{m^2}{4}\int_{\mathbb{R}^{2d}} dc_1dc_2\,e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\Big(c_2^2-\frac{d+2}{2}\Big)c_{2i}\Big[\Big(c_1^2-\frac{d+2}{2}\Big)c_{1i}+\Big(c_2^2-\frac{d+2}{2}\Big)c_{2i}\Big]$$

$$= \sigma^{d-1}\frac{\phi n^2}{\pi^d}\frac{m^2}{4}V_T^7\int_{\mathbb{R}^{2d}} dc_1dc_2\,e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\Big(c_2^2-\frac{d+2}{2}\Big)\Big[\Big(c_1^2-\frac{d+2}{2}\Big)(c_1\cdot c_2)+c_2^4-\frac{d+2}{2}c_2^2\Big]$$

$$= \sigma^{d-1}\frac{\phi n^2m^2V_T^7}{4\pi^d}\int_{\mathbb{R}^{2d}} dc_1dc_2\,e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\Big[\Big(c_1^2-\frac{d+2}{2}\Big)\Big(c_2^2-\frac{d+2}{2}\Big)(c_1\cdot c_2)+\Big(c_2^2-\frac{d+2}{2}\Big)^2c_2^2\Big]$$

$$= \sigma^{d-1}\frac{\phi n^2m^2V_T^7}{4\pi^d}\int_{\mathbb{R}^{2d}} d\underline{c}_1d\underline{c}_2\,e^{-c_1^2}e^{-c_2^2}\Big[(c_1^2+c_2^2)\Big(c_2^2-\frac{d+2}{2}\Big)^2c_2^2 - 2\Big(c_1^2-\frac{d+2}{2}\Big)\Big(c_2^2-\frac{d+2}{2}\Big)(c_1\cdot c_2)^2\Big]$$

$$= \sigma^{d-1}\frac{\phi n^2m^2V_T^7}{4\pi^d}\int_{\mathbb{R}^{2d}} d\underline{c}_1d\underline{c}_2\,e^{-c_1^2}e^{-c_2^2}\Big[(c_1^2c_2^2+c_2^4)\Big[c_2^4+\frac{(d+2)^2}{4}-(d+2)c_2^2\Big] - \frac{2}{d}c_1^2c_2^2\Big[c_1^2c_2^2-\frac{d+2}{2}c_1^2-\frac{d+2}{2}c_2^2+\frac{(d+2)^2}{4}\Big]\Big]$$

$$= \sigma^{d-1}\frac{\phi n^2m^2V_T^7}{4\pi^d}\int_{\mathbb{R}^{2d}} d\underline{c}_1d\underline{c}_2\,e^{-c_1^2}e^{-c_2^2}\Big[c_1^2c_2^6 + \underline{c_1^2c_2^2\frac{(d+2)^2}{4}} - c_1^2c_2^4(d+2) + c_2^8 + \frac{(d+2)^2}{4}c_2^4 - (d+2)c_2^6$$
$$-\frac{2}{d}c_1^4c_2^4 + \frac{2}{d}\frac{d+2}{2}c_1^4c_2^2 + \frac{2}{d}\frac{d+2}{2}c_1^2c_2^4 - \underline{\frac{2}{d}\frac{(d+2)^2}{4}c_1^2c_2^2}\Big]$$

$$= \sigma^{d-1}\frac{\phi n^2m^2V_T^7}{4\pi^d}\int_{\mathbb{R}^{2d}} d\underline{c}_1d\underline{c}_2\,e^{-c_1^2}e^{-c_2^2}\Big[c_1^2c_2^6 + c_2^8 + \frac{(d+2)^2}{4}c_2^4 - (d+2)c_2^6 - \frac{2}{d}c_1^4c_2^4 + c_1^2c_2^2\frac{(d+2)^2}{4}\frac{d-2}{d}$$
$$+ c_1^2c_2^4\Big\{-(d+2)+\frac{d+2}{d}+\frac{d+2}{d}\Big\}\Big]$$

$$= \sigma^{d-1}\frac{\varnothing n^2 m^2 V_T^7}{4\pi^d}\pi^d\Big[\frac{d}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2} + \frac{d+6}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2} + \frac{(d+2)^3}{8}\frac{d}{2} - (d+2)\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2} - \frac{2}{d}\frac{d+2}{2}\frac{d}{2}\frac{d+2}{2}\frac{d}{2} + \frac{d}{2}\frac{d}{2}\frac{(d+2)^2}{4}\frac{d-2}{d} + \frac{d}{2}\frac{d+2}{2}\frac{d}{2}\frac{2(d+2)-d(d+2)}{d}\Big]$$

$$= \sigma^{d-1}\frac{\varnothing n^2 m^2 V_T^7}{4\cancel{\pi^d}}\cancel{\pi^d}\frac{d}{2}\frac{d+2}{2}\Big[\frac{d}{2}\frac{d+4}{2} + \frac{d+6}{2}\frac{d+4}{2} + \frac{(d+2)^2}{4} - (d+2)\frac{d+4}{2} - \frac{\cancel{2}}{\cancel{d}}\frac{d+2}{\cancel{2}}\frac{\cancel{d}}{2} + \frac{\cancel{d}}{2}\frac{d+2}{2}\frac{d-2}{\cancel{d}} + \frac{\cancel{d}}{2}\frac{(d+2)(2-d)}{\cancel{d}}\Big]$$

$$= \sigma^{d-1}\frac{\varnothing n^2 m^2 V_T^7}{4}\frac{d}{2}\frac{d+2}{2}\frac{1}{2}\Big[\frac{d(d+4)}{2} + \frac{(d+6)(d+4)}{2} + \frac{(d+2)^2}{2} - (d+2)(d+4) - (d+2) + \frac{(d+2)(d-2)}{2} - (d+2)(d-2)\Big]$$

$$= \sigma^{d-1}\frac{\varnothing n^2 m^2 V_T^7}{4}\frac{d(d+2)}{2\cdot 8}\Big[(d+4)\{d + d+6 - 2(d+2)\} + (d+2)(\cancel{d}+2-\cancel{2}+\cancel{d}-\cancel{2}-\cancel{2d}+\cancel{4})\Big]$$

$$= \sigma^{d-1}\frac{\varnothing n^2 m^2 V_T^7}{8}\frac{d}{2}\frac{d+2}{2}\frac{1}{2}\Big[\underbrace{(d+4)\{\cancel{2d}+6-\cancel{2d}-4\} + 2(d+2)}_{=2(d+4)+2(d+2)=4(d+3)}\Big]$$

$$= \sigma^{d-1}\frac{\varnothing n^2 m^2 V_T^7}{4}\frac{d}{2}\frac{d+2}{2}(d+3)$$ ✓ (vérification : il y avait une erreur de calcul ; corrigée)

$$\frac{2m\beta^3}{d(d+2)nV_0}\int_{\mathbb{R}^d} d\underline{V}\, S_i(\underline{V}) L_a[\mathcal{M}S_i] = \frac{\cancel{2}m\beta^3}{\cancel{d(d+2)}\cancel{n}V_0}\sigma^{d-1}\frac{\varnothing n\cancel{2}m^2V_T^7}{4}\frac{\cancel{d}}{\cancel{2}}\frac{\cancel{d+2}}{2}(d+3)$$

$$= \frac{1}{\cancel{n}k_BT}\frac{d+2}{8}\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\frac{\sqrt{mk_BT}}{\cancel{\sigma^{d-1}}}\beta^3\cancel{\sigma^{d-1}}m^3\frac{\varnothing\cancel{n}V_T^7}{4}\frac{d+3}{2}$$

$$= \frac{1}{\cancel{k_BT}}\frac{d+2}{\cancel{8}}\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\cancel{\sqrt{mk_BT}}\,\cancel{\beta^3m^3}\frac{\varnothing}{4}\sqrt{\frac{2}{\cancel{\beta m}}}\frac{\cancel{8}}{\cancel{\beta^3m^3}}\frac{d+3}{2}$$

$$= \varnothing\frac{(d+2)(d+3)}{8}\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}$$ ✓

Calcul de ν_K^{*c} : on reprend une étape intermédiaire du gaz de Maxwell :

$$\int_{\mathbb{R}^d} d\underline{V}\, S_i(\underline{V}) L_c[\mathcal{M}S_i] = -\sigma^{d-1}\frac{\varnothing V_T^7}{S_d}\frac{m^2n^2}{4\pi^d}\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}(c_1^2+c_2^2-2c_1\cdot c_2)\left(c_2^2-\frac{d+2}{2}\right)c_{2i}\int d\hat{\sigma}\,(b-1)\Big[c_1^2c_{1i}+c_2^2c_{2i}-\frac{d+2}{2}(c_{1i}+c_{2i})\Big]$$

$$(b-1)[\dots] = (g\cdot\hat{\sigma})^2(c_{1j}+c_{2j})[\delta_{ij}+2\hat{\sigma}_i\hat{\sigma}_j] - (g\cdot\hat{\sigma})\hat{\sigma}_j[c_1^2\delta_{ij}-c_2^2\delta_{ij}+2c_{1i}c_{1j}-2c_{2i}c_{2j}]$$ ✓

Les intégrations faites (cf. gaz Maxwell), il vient :

$$c_{2i}(c_{1i}+c_{2i})\beta_2(c_1^2+c_2^2-2c_1\cdot c_2) = \beta_2\big[(c_1^2+c_2^2-2c_2^2)(c_1\cdot c_2) - 2(c_1\cdot c_2)^2 + c_1^2c_2^2 + c_2^4\big]$$

$$= \beta_2\Big[(c_1^2-c_2^2)(c_1\cdot c_2) - \frac{2}{d}c_1^2c_2^2 + c_1^2c_2^2 + c_2^4\Big]$$

0 : contributions s'annulent : $c_1\leftrightarrow c_2$ NON : il y a un poids différent pour c_1^2 et c_2^2 !

$$= \beta_2\Big[\cancel{(c_1^2-c_2^2)}\underbrace{(c_1\cdot c_2)}_{\text{antisym.}} + \frac{d-2}{d}c_1^2c_2^2 + c_2^4\Big]$$ ✓

$$c_{2i}\,2(c_{1j}+c_{2j})\frac{\beta_2}{d+2}(2g_ig_j+g^2\delta_{ij}) = \frac{2\beta_2}{d+2}\Big[2\Big\{(c_1\cdot c_2)c_1^2 - \cancel{\tfrac{1}{d}c_1^2c_2^2} - c_1^2c_2^2 + \cancel{c_2^2(c_1\cdot c_2)} + \cancel{\tfrac{1}{d}c_1^2c_2^2} - \cancel{c_2^2(c_1\cdot c_2)} - (c_1\cdot c_2)c_2^2 + c_2^4\Big\}$$
$$+ c_1^2c_2^2 + c_2^4 - 2c_2^2(c_1\cdot c_2) + (c_1^2+c_2^2)(c_1\cdot c_2) - \frac{2}{d}c_1^2c_2^2\Big]$$ ✓

$$= \frac{2\beta_2}{d+2}\Big[2c_1^2(c_1\cdot c_2) - 2c_1^2c_2^2 - 2c_2^2(c_1\cdot c_2) + 2c_2^4 + c_1^2c_2^2 + c_2^4 - 2c_2^2(c_1\cdot c_2) + c_1^2(c_1\cdot c_2) + c_2^2(c_1\cdot c_2) - \frac{2}{d}c_1^2c_2^2\Big]$$

$$= \frac{2\beta_2}{d+2}\Big[3c_2^4 - c_1^2c_2^2\frac{d+1}{d}2 + (c_1\cdot c_2)\{2c_1^2-2c_2^2-2c_2^2+c_1^2+c_2^2\}\Big]$$

0 : contributions égales : $c_1\leftrightarrow c_2$ NON.

$$= \frac{2\beta_2}{d+2}\Big[3c_2^4 - c_1^2c_2^2\frac{d+1}{d}2 + 3\cancel{(c_1^2-c_2^2)}(c_1\cdot c_2)\Big]$$

$$= \frac{2\beta_2}{d+2}\Big[3c_2^4 - c_1^2c_2^2\frac{d+1}{d}2\Big] + 3(c_1^2-c_2^2)(c_1\cdot c_2)$$

$$-c_{2i}\beta_2 g_j(c_1^2\delta_{ij}-c_2^2\delta_{ij}+2c_{1i}c_{1j}-2c_{2i}c_{2j}) = -\beta_2\big[c_1^2(c_1\cdot c_2) - c_2^2(c_1\cdot c_2) + 2c_1^2(c_1\cdot c_2) - 2c_2^2(c_1\cdot c_2) - c_2^2c_1^2 + c_2^4 - 2(c_1\cdot c_2)^2 + 2c_2^4\big]$$

$$= -\beta_2\Big[3c_2^4 - c_1^2c_2^2\frac{d+2}{d} + (c_1\cdot c_2)\{c_1^2-c_2^2+2c_1^2-2c_2^2\}\Big]$$

NON.

$$= -\beta_2\Big[3c_2^4 - c_1^2c_2^2\frac{d+2}{d} + 3\cancel{(c_1^2-c_2^2)}^{\,0}(c_1\cdot c_2)\Big]$$

$$= -\beta_2\Big[3c_2^4 - c_1^2c_2^2\frac{d+2}{d}\Big] + 3(c_1^2-c_2^2)(c_1\cdot c_2)$$ ✓

Ainsi :

$$\int_{\mathbb{R}^d} dv\, S_i(\underline{v})\, L_c[MS_i] = -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4\pi^d}\int_{\mathbb{R}^{2d}} d\underline{c_1}\,d\underline{c_2}\, e^{-c_1^2}e^{-c_2^2}\left(c_1^2+c_2^2-\cancelto{0}{2c_1\cdot c_2}\right)\left(c_2^2-\frac{d+2}{2}\right)\beta_2\left[c_2^4+c_1^2c_2^2\frac{d-2}{d}+\left(3c_2^4-c_1^2c_2^2\frac{d+2}{d}\right)\overbrace{\left(-1+\frac{2}{d+2}\right)}^{=\frac{-d}{d+2}}\right]$$ (6) (V)

+ Impair (6b)

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4\pi^d}\beta_2\int_{\mathbb{R}^{2d}} d\underline{c_1}\,d\underline{c_2}\, e^{-c_1^2}e^{-c_2^2}\left(c_1^2+c_2^2\right)\left[c_2^6\,\overbrace{\frac{(-2)(d-1)}{d+2}}^{=-2\frac{d-1}{d+2}}+c_1^2c_2^4\frac{d^2+2(d-2)}{2d}-c_2^4\overbrace{\frac{d+2}{2}\frac{(-2)(d-1)}{d+2}}^{=-2\frac{d+2}{2}\frac{d-1}{d+2}}\right.$$
$$\left. - c_1^2c_2^2\frac{d+2}{2}\frac{d^2+2(d-2)}{2d}\right]$$

ok iden

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4\pi^d}\beta_2\int_{\mathbb{R}^{2d}} d\underline{c_1}\,d\underline{c_2}\, e^{-c_1^2}e^{-c_2^2}\left[-2\frac{d-1}{d+2}c_1^2c_2^6+c_1^4c_2^4\frac{d^2+2(d-2)}{2d}+2\frac{d+2}{2}\frac{d-1}{d+2}c_1^2c_2^4-c_1^4c_2^2\frac{d+2}{2}\frac{d^2+2(d-2)}{2d}\right.$$
$$\left.-2\frac{d-1}{d+2}c_2^8+c_1^2c_2^6\frac{d^2+2(d-2)}{2d}+2\frac{d+2}{2}\frac{d-1}{d+2}c_2^6-c_1^2c_2^4\frac{d+2}{2}\frac{d^2+2(d-2)}{2d}\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4\pi^d}\beta_2\,\pi^d\left[-2\frac{d-1}{d+2}\frac{d}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}+\frac{d+2}{2}\frac{d}{2}\frac{d+2}{2}\frac{d}{2}\frac{d^2+2(d-2)}{2d}+2\frac{d+2}{2}\frac{d-1}{d+2}\frac{d}{2}\frac{d+2}{2}\frac{d}{2}\right.$$
$$-\frac{d+2}{2}\frac{d}{2}\frac{d}{2}\frac{d+2}{2}\frac{d^2+2(d-2)}{2d}$$
$$-2\frac{d-1}{d+2}\frac{d+6}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}+\frac{d}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}\frac{d^2+2(d-2)}{2d}+2\frac{d+2}{2}\frac{d-1}{d+2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}$$
$$\left.-\frac{d}{2}\frac{d+2}{2}\frac{d}{2}\frac{d+2}{2}\frac{d^2+2(d-2)}{2d}\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4\cancel{\pi^d}}\beta_2\cancel{\pi^d}\frac{d+2}{2}\frac{d}{2}\left[-\cancel{2}\frac{d-1}{d+2}\frac{d}{\cancel{2}}\frac{d+4}{2}+\frac{d+2}{2}\frac{\cancel{d}}{2}\frac{d^2+2(d-2)}{2\cancel{d}}+2\frac{d+2}{2}\frac{d}{2}\frac{d-1}{d+2}-\frac{d+2}{2}\frac{\cancel{d}}{2}\frac{d^2+2(d-2)}{\cancel{d}}\right.$$
$$\left.-\cancel{2}\frac{d-1}{d+2}\frac{d+6}{\cancel{2}}\frac{d+4}{2}+\frac{\cancel{d}}{2}\frac{d+4}{2}\frac{d^2+2(d-2)}{2\cancel{d}}+2\frac{d+2}{2}\frac{d-1}{d+2}\frac{d+4}{2}\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d+2}{2}\frac{d}{2}\left[\frac{1}{2}\frac{d-1}{d+2}\Big(-d(d+4)+d(d+2)-(d+4)(d+6)+(d+4)(d+2)\Big)\right.$$
$$\left.+\frac{d^2+2(d-2)}{8}\Big(d+2-2(d+2)+d+4\Big)\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d+2}{2}\frac{d}{2}\frac{1}{2}\left[\frac{d-1}{d+2}\left\{(d+4)(-d\cancel{-d}-6\cancel{+d}+2)+d(d+2)\right\}+\frac{d^2+2(d-2)}{4}\Big(-d-2+d+4\Big)\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d+2}{2}\frac{d}{2}\frac{1}{2}\left[\frac{d-1}{d+2}\left\{(d+4)(-d-4)+d(d+2)\right\}+\frac{d^2+2(d-2)}{2}\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{\cancel{d+2}}{2}\frac{d}{2}\frac{1}{2}\frac{1}{2(\cancel{d+2})}\left[2(d-1)\left\{-(d+4)^2+d(d+2)\right\}+(d+2)\left\{d^2+2(d-2)\right\}\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d}{16}\left[2(d-1)\{\underbrace{\cancel{d^2}+2d-\cancel{d^2}-16-8d}_{=-6d-16}\}+d^3+2d^2\cancel{-4d}+2d^2\cancel{+4d}-8\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d}{16}\left[-12d^2-32d+12d+32+d^3+4d^2-8\right]$$

$$= -\sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d}{16}\left[d^3-8d^2-20d+24\right]$$

erreur : $8(d-1)(d+3)$

$$= \sigma^{d-1}\frac{\phi V_T^7}{S_d}\frac{m^2n^2}{4}\beta_2\frac{d}{16}\left[\cancel{-24+20d+8d^2}-d^3\right] + \mathrm{I}$$ ✓

$$\frac{2m\beta^3}{d(d+2)nV_0}\int_{\mathbb{R}^d} dv\, S_i(\underline{v})\, L_c[MS_i] = \frac{2\cancel{m\beta^3}}{\cancel{d}(d+2)\cancel{n}V_0}\sigma^{d-1}\phi\frac{\Gamma(d/2)}{\cancel{2}\pi^{d/2}}\sqrt{\frac{2}{\beta m}}^{\,7}\frac{\cancel{d}}{\cancel{\beta^3 m^3}}\frac{\cancel{m^2n^2}}{\cancel{4}}2\pi^{(d-1)/2}\frac{\Gamma\left(\frac{1+2}{2}\right)}{\Gamma\left(\frac{d+2}{2}\right)}\frac{\cancel{d}}{16}[\dots]$$

$$= \frac{2}{\cancel{d+2}}\frac{1}{\cancel{\dots}}\frac{\cancel{d+2}}{8}\frac{\Gamma(d/2)}{\cancel{\pi^{(d-1)/2}}}\frac{\sqrt{\cancel{\dots}}}{\cancel{\sigma^{d-1}}}\cancel{\sigma^{d-1}}\phi\frac{\cancel{\dots}}{\pi^{d/2}}\sqrt{\frac{2}{\cancel{\beta m}}}\cancel{\dots}\cancel{2\pi^{(d-1)/2}}\frac{1}{\cancel{2}}\sqrt{\pi}\frac{1}{\frac{d}{2}\cancel{\Gamma(d/2)}}\frac{1}{16}[\dots]$$

$$= \frac{\cancel{2}}{\cancel{8}2}\Gamma(d/2)\,\phi\frac{\sqrt{2}}{\pi^{d/2}}\sqrt{\pi}\frac{\cancel{2}}{16}\frac{1}{d}[\dots]$$

$$= \phi\frac{1}{32d}\left[\cancel{-16+20d+8d^2}-d^3\right]\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}$$ ✓

$8(d-1)(d+3)+8(d-1)$

$= 8(d-1)(d+4)$

Terme impar en plus:

$$I = -\sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4\pi^d} \int_{\mathbb{R}^d} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (-2)\, c_1\cdot c_2 \left(c_2^2 - \frac{d+2}{2}\right) \beta_2 \left[(c_1^2 - c_2^2)(c_1\cdot c_2) + \frac{6}{d+2}(c_1^2 - c_2^2)(c_1\cdot c_2) - 3(c_1^2-c_2^2)(c_1\cdot c_2)\right]$$

$$= -\sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{1}{\pi^d}(-2) \int_{\mathbb{R}^d} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} \underbrace{(c_1\cdot c_2)^2}_{\to 1/d\, c_1^2 c_2^2} \left(c_2^2 - \frac{d+2}{2}\right) \Bigg[\overbrace{\frac{6}{d+2} - 2}^{\frac{6-2d-4}{d+2} = \frac{-2d+2}{d+2}} \Bigg] (c_1^2 - c_2^2)$$

$$= \sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{2}{\pi^d} \int_{\mathbb{R}^d} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} \frac{1}{d} c_1^2 c_2^2 \left(c_2^2 - \frac{d+2}{2}\right)(-2)\frac{d-1}{d+2}(c_1^2 - c_2^2)$$

$$= -\sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{4}{\pi^d} \frac{d-1}{d+2} \frac{1}{d} \int_{\mathbb{R}^d} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} \underbrace{(c_1^4 c_2^2 - c_1^2 c_2^4)\left(c_2^2 - \frac{d+2}{2}\right)}_{= c_1^4 c_2^4 - \frac{d+2}{2} c_1^4 c_2^2 - c_1^2 c_2^6 + c_1^2 c_2^4 \frac{d+2}{2}}$$

$$= -\sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{4}{\cancel{\pi^d}} \frac{d-1}{d(d+2)} \cancel{\pi^d} \left[\frac{d+2}{2}\frac{d}{2}\frac{d+2}{2}\frac{d}{2} - \frac{d}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2} \right]$$

$$= -\sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{4(d-1)}{\cancel{d(d+2)}} \frac{d^{\cancel{2}}\cancel{(d+2)}}{16} \Big[\underbrace{\cancel{d}+2 - \cancel{d} - 4}_{=-2} \Big]$$

$$= \sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{d}{2}(d-1)$$

$$= \sigma^{d-1} \frac{\phi V_T^7}{S_d} \frac{m^2 n^2}{4} \beta_2 \frac{d}{16}\, 8(d-1)$$

$$I = \psi \frac{1}{\pi^d}\int dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2)\left(c_2^2-\frac{d+2}{2}\right)\left[c_2^4 + c_1^2c_2^2\frac{d-2}{d} + \underbrace{\frac{2-d-2}{d+2}}_{-\frac{d}{d+2}}\left(3c_2^4 - c_1^2c_2^2\frac{d+2}{d}\right)\right]$$

$$c_2^4 + c_1^2c_2^2\frac{d-2}{d} - \frac{3d}{d+2}c_2^4 + \frac{\cancel{d+2}\,\cancel{d}}{\cancel{d}\,\cancel{d+2}}\, c_1^2c_2^2$$

$$c_2^4 \underbrace{\frac{d+2-3d}{d+2}}_{\underbrace{\frac{-2d+2}{d+2}}_{-2\frac{d-1}{d+2}}} + c_1^2c_2^2 \underbrace{\frac{d-2+d}{d}}_{2\frac{d-1}{d}}$$

$$-2\frac{d-1}{d+2}c_2^4 + 2\frac{d-1}{d}c_1^2c_2^2 \quad \checkmark$$

$$= \psi\frac{2(d-1)}{\pi^d}\int dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2)\left(c_2^2-\frac{d+2}{2}\right)\left(-\frac{1}{d+2}c_2^4 + \frac{1}{d}c_1^2c_2^2\right)$$

$$\cancel{c_1^2c_2^2 - \frac{d+2}{2}c_1^2 + c_2^4 - \frac{d+2}{2}c_2^2}$$

$$\cancel{-\frac{1}{d+2}c_1^2c_2^6 + \frac{1}{d}c_1^4c_2^4}$$

$$\cancel{-\frac{d+2}{2}\left(-\frac{1}{d+2}\right)}$$

$$-\frac{1}{d+2}c_2^6 + \frac{1}{d}c_1^2c_2^4 + \frac{\cancel{d+2}}{2}\frac{1}{\cancel{d+2}}c_2^4 - \frac{d+2}{2d}c_1^2c_2^2$$

$$-\frac{1}{d+2}c_1^2c_2^6 + \frac{1}{d}c_1^4c_2^4 + \frac{1}{2}c_1^2c_2^4 - \frac{d+2}{2d}c_1^4c_2^2$$

$$-\frac{1}{d+2}c_2^8 + \frac{1}{d}c_1^2c_2^6 + \frac{1}{2}c_2^6 - \frac{d+2}{2d}c_1^2c_2^4$$

$$= \frac{1}{d}c_1^4c_2^4 - \frac{1}{d+2}c_2^8 + \frac{1}{2}c_2^6 + c_1^2c_2^6\underbrace{\left[\frac{\cancel{d}+2-\cancel{d}}{d(d+2)}\right]}_{2/d(d+2)}$$

$$+ c_1^2c_2^4\underbrace{\left[\frac{\cancel{d}\cancel{-d}-2-d-2}{2d}\right]}_{-\frac{d+4}{2d}}$$

$$= \varphi \frac{2(d-1)}{\pi^d} \int dc_1 dc_2 e^{-c_1^2} e^{-c_2^2} \left[\frac{1}{d} c_1^4 c_2^4 - \frac{1}{d+2} c_2^8 + \frac{1}{2} c_2^6 + \frac{2}{d(d+2)} c_1^2 c_2^6 - \frac{d+4}{2d} c_1^2 c_2^4 \right]$$

$$= \varphi \frac{2(d-1)}{\cancel{\pi^d}} \left[\frac{1}{\cancel{d}} \frac{d+2}{2} \frac{d}{2} \frac{d+2}{2} \frac{\cancel{d}}{2} - \frac{1}{\cancel{d+2}} \frac{d+6}{2} \frac{d+4}{2} \frac{\cancel{d+2}}{2} \frac{d}{2} + \frac{1}{2} \frac{d+4}{2} \frac{d+2}{2} \frac{d}{2} + \frac{2}{\cancel{d(d+2)}} \frac{\cancel{d}}{2} \frac{d+4}{2} \frac{\cancel{d+2}}{2} \frac{d}{2} - \frac{d+4}{2\cancel{d}} \frac{d}{2} \frac{d+2}{2} \frac{\cancel{d}}{2} \right]$$

$$= \varphi \frac{d(d-1)}{8} \left[(d+2)^2 \underbrace{- (d+6)(d+4) + (\cancel{d+2})(d+4) + 2(d+4)}_{= (d+4)\left[\underbrace{2-d-6}_{=-d-4}\right]} - (\cancel{d+2})(d+4) \right]$$

$$= \varphi \frac{d(d-1)}{8} \left[\underbrace{(d+2)^2 - (d+4)^2}_{\underbrace{\cancel{d^2} + 4 + 4d - \cancel{d^2} - 16 - 8d}_{= -4d - 12 = -4(d+3)}} \right]$$

$$= -\varphi \frac{d(d-1)(d+3)}{2}$$

Ok !!!

$$\int \ldots = \sigma^{d-1} \frac{\varphi V_T^7}{\Omega} \frac{m^2 n^2}{4} \beta_2 \underbrace{\frac{d}{16} \left[8(d-1)(d+3) + 8(d-1) \right]}_{= \frac{d(d-1)}{2} (d+3+1) = \frac{d}{2}(d-1)(d+4)}$$

Calcul de $V_K^{*a'}$:

$$V_K^{*a'} = -\frac{2m\beta^3}{d(d+2)nv_0}\int_{\mathbb{R}^d} dV\, S_i(V)\,\Omega[MS_i]$$

$$= \frac{2m\beta^3}{d(d+2)\,nv_0} K_{ij}\int_{\mathbb{R}^d} dV\, S_i(V)\,\frac{\partial f^{(0)}}{\partial V_j}$$

Ce calcul est identique à celui de Maxwell, car $a_2=0$ (la différence entre Maxwell et VHP est contenue dans K_{ij}, mais on trouve que $V_K^{*a'}=0$ indép. de K_{ij}), et donc $V_K^{*a'}=0$.

Calcul de $V_\eta^{*a'}$: à nouveau, on a $V_\eta^{*a'}=0$ car la seule différence avec le calcul pour Maxwell est la valeur de K_{ij} et celle de L_{ij}. Mais la nullité de ce coefficient est établie indép. de K_{ij} et L_{ij}, donc $V_\eta^{*a'}=0$.

Conclusion :

$$\boxed{\begin{aligned} V_\eta^* &= p\,\phi\,\frac{(d+2)^2}{8}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} + (1-p)\,\phi\,\frac{(d+2)(d+4)}{16}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} = \phi\,\frac{\Gamma(\frac{d}{2})\sqrt{2}}{4\pi^{(d-1)/2}}\left[p\,\frac{(d+2)^2}{2} + (1-p)\,\frac{(d+2)(d+4)}{4}\right] \\ V_K^* = V_\mu^* &= p\,\phi\,\frac{(d+2)(d+3)}{8}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} + (1-p)\,\phi\,\frac{d^3-8d^2-20d+24}{32d}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} = \phi\,\frac{\Gamma(\frac{d}{2})\sqrt{2}}{4\pi^{(d-1)/2}}\left[p\,\frac{(d+2)(d+3)}{2} + (1-p)\,\cancel{\frac{-24+20d+8d^2-d^3}{8d}}\right]\end{aligned}}$$

$\frac{(d-1)(d+4)}{d}$

Distribution à l'ordre 1 : idem. Stabilité linéaire : calcul des taux de déclin au premier ordre.

Que vaut ϕ ? C'est la fréquence de collision (adimensionnalisée), qui dépend du système considéré. On peut donc choisir une autre valeur que celle pour le modèle de Maxwell. On peut choisir ϕ de sorte que $V_\eta^*(p=0)=1$, ou bien essaye une dérivation à la Santos. Si on choisit $V_\eta^*(p=0)=1$, alors $\phi = \phi_{Maxwell}\, 4/(d+2)(d+4)$; $\phi_{Maxwell} = \frac{4}{\sqrt{2}}\frac{\pi^{(d-1)/2}}{\Gamma(d/2)}$.

mieux : $\eta(p=0)=1$ (c'est la même condition)

$$\xi_n^{(0)*} = \frac{1}{\cancel{\ldots}}\,\frac{d+2}{8}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\,\frac{\cancel{\sqrt{\ldots}}}{\cancel{\ldots}}\,\cancel{\ldots}\,\phi\sqrt{\frac{2}{\cancel{\beta}}}\,d = d\,\frac{d+2}{8}\,\phi\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} = d\,\frac{\cancel{d+2}}{8}\,\frac{4\cdot 4}{\cancel{(d+2)}(d+4)} = \frac{2d}{d+4}$$

$$\xi_T^{(0)*} = \frac{1}{d}\,\xi_n^{(0)*} = \frac{2}{d+4}$$

Calcul des taux de déclin au premier ordre :

$$\omega[Af^{(0)}, Bf^{(1)}] = -\sigma^{d-1}\frac{\phi}{\cancel{V_T}}\frac{\beta^3 n}{\pi^d}\frac{2m}{d+2}\left(\kappa\nabla_i T + \mu\nabla_i n\right)\frac{m}{2}V_T^2\cancel{V_T}V_T^2\, I_1 - \sigma^{d-1}\frac{\phi}{\cancel{V_T}}\frac{\beta^3 n}{\pi^d}\frac{\eta}{\beta}\nabla_j u_i\, m V_T^2 V_T^{\cancel{*}}\, I_2$$

$$I_1 = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, c_{12}^2\, A(V_T c_2)\, B(V_T c_1)\, e^{-c_1^2} e^{-c_2^2}\left(c_1^2 - \frac{d+2}{2}\right)c_{1i}$$

$$I_2 = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, c_{12}^2\, A(V_T c_2)\, B(V_T c_1)\, e^{-c_1^2} e^{-c_2^2}\left(c_{1i}c_{1j} - \frac{1}{d}c_1^2\delta_{ij}\right)$$

Simplification de $\omega[Af^{(0)}, Bf^{(1)}]$: (cf. 7b)

$$\omega[Af^{(0)}, Bf^{(1)}] = -\phi\,\frac{nV_T}{\pi^d}\,\frac{d}{d-1}\,\frac{d+2}{8}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\left(\kappa^*\frac{1}{T}\nabla_i T + \mu^*\frac{1}{n}\nabla_i n\right)I_1 - \phi\,\frac{n}{\pi^d}\,\frac{d+2}{4}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\nabla_j u_i\, I_2$$

Calcul des différents termes :

$$\xi_n^{(1)} = \frac{2}{n}\,\omega[f^{(0)}, f^{(1)}]$$

$$I_1^{1,1} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\,(c_1^2+c_2^2-2c_1\cdot c_2)\,e^{-c_1^2}e^{-c_2^2}\left(c_1^2 c_{1i} - \frac{d+2}{2}c_{1i}\right)$$

$$= \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}\left[c_1^4\cancel{c_{1i}}^{0} - \frac{d+2}{2}c_1^2\cancel{c_{1i}}^{0} + c_2^2c_1^2\cancel{c_{1i}}^{0} - \frac{d+2}{2}c_2^2\cancel{c_{1i}}^{0} - 2(c_1\cdot\cancel{c_2})^{0}c_1^2c_{1i} + 2(c_1\cdot\cancel{c_2})^{0}\frac{d+2}{2}c_{1i}\right]$$

$$= 0$$

$$I_2^{1,1} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\,(c_1^2+c_2^2-2c_1\cdot c_2)\,e^{-c_1^2}e^{-c_2^2}\left(c_{1i}c_{1j} - \frac{1}{d}c_1^2\delta_{ij}\right)$$

$$= \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}\left[c_1^2c_{1i}c_{1j} - \frac{1}{d}c_1^4\delta_{ij} + c_2^2c_{1i}c_{1j} - \frac{1}{d}c_1^2c_2^2\delta_{ij} - 2(c_1\cdot\cancel{c_2})^{0}c_{1i}c_{1j} + \frac{2}{d}c_1^2(c_1\cdot\cancel{c_2})^{0}\delta_{ij}\right]$$

$$= \pi^d\left[\frac{d+2}{2d}\,\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\delta_{ij} - \frac{1}{d}\,\frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)}\delta_{ij} + \frac{d}{2d}\delta_{ij}\,\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} - \frac{1}{d}\,\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\,\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\right]$$

$$= \delta_{ij}\pi^d\left[\frac{d+2}{2\cancel{d}}\frac{\cancel{d}}{2} - \frac{1}{\cancel{d}}\frac{d+2}{2}\frac{\cancel{d}}{2} + \frac{1}{2}\frac{d}{2} - \frac{1}{\cancel{d}}\frac{\cancel{d}}{2}\frac{d}{2}\right]$$

$$= 0$$

$$\omega[Af^{(0)}, Bf^{(1)}] = -\int_{\mathbb{R}^d} d\underline{v}_1\, J_a[Af^{(0)}, Bf^{(1)}]$$

$$= \sigma^{d-1}\frac{\phi}{v_T}\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\; v_{12}^2\; A(v_2)\,B(v_1)\; f^{(0)}(v_2)\, f^{(1)}(v_1)$$

$$= \sigma^{d-1}\frac{\phi}{v_T}\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\; v_{12}^2\; A(v_2)B(v_1)\,\frac{n}{v_T^d}\frac{1}{\pi^{d/2}}e^{-v_2^2/v_T^2}\left[-\frac{\beta^3}{n}\;\frac{n}{v_T^d}\frac{1}{\pi^{d/2}}e^{-v_1^2/v_T^2}\right]\left[\frac{2m}{d+2}S_i(v_1)\left(\kappa\nabla_i T+\mu\nabla_i n\right)+\frac{\eta}{\beta}D_{ij}(v_1)\nabla_j u_i\right]$$

$$= -\sigma^{d-1}\frac{\phi}{v_T}\,\frac{n}{v_T^d}\,\frac{1}{\pi^{d/2}}\,\frac{\beta^3}{n}\,\frac{n}{v_T^d}\,\frac{1}{\pi^{d/2}}\int_{\mathbb{R}^{2d}} d\underline{v}_1 d\underline{v}_2\; v_{12}^2\, A(v_2)B(v_1)\,e^{-v_2^2/v_T^2}e^{-v_1^2/v_T^2}\left[\frac{2m}{d+2}\,\frac{m}{2}\left(v_1^2-\frac{d+2}{2}v_T^2\right)v_{1i}\left(\kappa\nabla_i T+\mu\nabla_i n\right)\right.$$
$$\left.+\frac{\eta}{\beta}\,m\left(v_{1i}v_{1j}-\frac{1}{d}v_1^2\delta_{ij}\right)\nabla_j u_i\right]$$

$$= -\sigma^{d-1}\frac{\phi}{\cancel{v_T^{2d}}}\,\frac{n^{\cancel{2}}}{\cancel{n}}\,\frac{\beta^3}{\cancel{n}}\,\frac{1}{\pi^d}\,\cancel{v_T^{2d}}\,v_T^{\cancel{1}}\,v_T^2\int_{\mathbb{R}^{2d}} d\underline{c}_1 d\underline{c}_2\; c_{12}^2\, A(v_Tc_1)\,B(v_Tc_2)\,e^{-c_1^2}e^{-c_2^2}\left[\frac{2m}{d+2}\,\frac{m}{2}v_T\left(c_1^2-\frac{d+2}{2}\right)c_{1i}\left(\kappa\nabla_i T+\mu\nabla_i n\right)\right.$$
$$\left.+\frac{\eta}{\beta}\,m\left(c_{1i}c_{1j}-\frac{1}{d}c_1^2\delta_{ij}\right)\nabla_j u_i\right]$$

$$= -\sigma^{d-1}\phi\, n\,\beta^3 v_T^3\frac{1}{\pi^d}\;\frac{2m}{d+2}\;\frac{m}{2}v_T\underbrace{\int_{\mathbb{R}^{2d}} dc_1 dc_2\; c_{12}^2\, A(v_Tc_1)\,B(v_Tc_2)\,e^{-c_1^2}e^{-c_2^2}\left(c_1^2-\frac{d+2}{2}\right)c_{1i}}_{=I_1}\;\left(\kappa\nabla_i T+\mu\nabla_i n\right)$$

$$-\sigma^{d-1}\phi\, n\,\beta^3 v_T^3\frac{1}{\pi^d}\;\frac{m}{\beta}\underbrace{\int_{\mathbb{R}^{2d}} dc_1 dc_2\; c_{12}^2\, A(v_Tc_1)\,B(v_Tc_2)\,e^{-c_1^2}e^{-c_2^2}\left(c_{1i}c_{1j}-\frac{1}{d}c_1^2\delta_{ij}\right)}_{=I_2}\eta\nabla_j u_i$$

$$= -\sigma^{d-1}\phi\, n\,\beta^3 v_T^4\frac{m^2}{d+2}\frac{1}{\pi^d}\left(\kappa\nabla_i T+\mu\nabla_i n\right)I_1-\sigma^{d-1}\phi\, n\,\beta^2\,v_T^3\,m\,\frac{1}{\pi^d}\,\eta\nabla_j u_i\, I_2$$

$$= -\sigma^{d-1}\phi\, n\,\beta^{\cancel{3}}\frac{4}{\cancel{\beta^2 m^2}}\,\frac{\cancel{m^2}}{d+2}\,\frac{1}{\pi^d}\left(\kappa_0\kappa^*\nabla_i T+\frac{T\kappa_0}{n}\mu^*\nabla_i n\right)I_1-\sigma^{d-1}\phi\, n\,\beta^{\cancel{2}}\frac{2}{\cancel{\beta m}}\sqrt{\frac{2}{\beta m}}\,\frac{\cancel{m}}{\pi^d}\,\eta_0\eta^*\nabla_j u_i\, I_2$$

$$= -\cancel{\sigma^{d-1}}\phi\, n\,\cancel{\beta}\,4\,\frac{1}{\cancel{d+2}}\,\frac{1}{\pi^d}\left[\frac{d(d+2)}{2(d-1)}\,\frac{\cancel{k_B}}{m}\,\frac{\cancel{d+2}}{8}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\,\frac{\sqrt{mk_BT}}{\cancel{\sigma^{d-1}}}\,\kappa^*\frac{1}{T}\nabla_i T+\frac{\cancel{T}}{\cancel{T}}\,\frac{d(d+2)}{2(d-1)}\,\frac{\cancel{k_B}}{m}\,\frac{\cancel{d+2}}{8}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\,\frac{\sqrt{mk_BT}}{\cancel{\sigma^{d-1}}}\,\mu^*\frac{1}{n}\nabla_i n\right]I_1$$
$$-\cancel{\sigma^{d-1}}\phi\, n\,\cancel{\beta}\,2\sqrt{\frac{2}{\cancel{\beta m}}}\,\frac{1}{\pi^d}\,\frac{d+2}{8}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\,\frac{\cancel{\sqrt{mk_BT}}}{\cancel{\sigma^{d-1}}}\,\eta^*\nabla_j u_i\, I_2$$

$$= -\phi\, n\,\cancel{4}\frac{1}{\pi^d}\,\frac{d}{\cancel{2}}\frac{(d+2)}{d-1}\,\frac{1}{\sqrt{m}}\,\frac{1}{\cancel{8}_4}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\sqrt{\cancel{m}k_BT}\left(\kappa^*\frac{1}{T}\nabla_i T+\mu^*\frac{1}{n}\nabla_i n\right)I_1-\phi\, n\, 2\sqrt{2}\,\frac{1}{\pi^d}\,\frac{d+2}{8}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\,\eta^*\nabla_j u_i\, I_2$$

$$= -\phi\, n\,\frac{1}{\pi^d}\,\frac{d(d+2)}{d-1}\,\frac{1}{4}\,\underbrace{\sqrt{\frac{2}{\beta m}}}_{=v_T}\,\underbrace{\frac{1}{\sqrt{2}}}_{=\sqrt{2}/2}\,\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}\left(\kappa^*\frac{1}{T}\nabla_i T+\mu^*\frac{1}{n}\nabla_i n\right)I_1-\phi\,\frac{n}{\pi^d}\,\frac{d+2}{4}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\,\nabla_j u_i\, I_2$$

$$= -\phi\,\frac{n v_T}{\pi^d}\,\frac{d}{d-1}\,\frac{d+2}{8}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\left(\kappa^*\frac{1}{T}\nabla_i T+\mu^*\frac{1}{n}\nabla_i n\right)I_1-\phi\,\frac{n}{\pi^d}\,\frac{d+2}{4}\,\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\,\nabla_j u_i\, I_2 \qquad \text{(ok, vérifié)}$$

$$\Longrightarrow \boxed{\xi_n^{(1)} = 0}$$

$$\xi_T^{(1)} = \underbrace{-\xi_n^{(1)}}_{=0} + \frac{m}{nk_BTd}\,\omega[f^{(0)}, V^2 f^{(1)}] + \frac{m}{nk_BTd}\,\omega[V^2 f^{(0)}, f^{(1)}]$$

$$I_1^{1,V_1^2} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, (c_1^2+c_2^2-2c_1\cdot c_2)\, e^{-c_1^2} e^{-c_2^2}\, V_T^2 c_1^2 \left(c_1^2 - \frac{d+2}{2}\right) c_{1i}$$

$$= V_T^2 \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2-2c_1\cdot c_2)\left(c_1^4 c_{1i} - \frac{d+2}{2} c_1^2 c_{1i}\right)$$

$$= V_T^2 \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\Big[c_1^6 \cancel{c_{1i}}^0 - \frac{d+2}{2} c_1^4 \cancel{c_{1i}}^0 + c_1^4 c_2^2 \cancel{c_{1i}}^0 - \frac{d+2}{2} c_1^2 c_2^2 \cancel{c_{1i}}^0 - 2(c_1\cdot \cancel{c_2})^0 c_1^4 c_{1i} + 2\frac{d+2}{2} c_1^2 (c_1\cdot\cancel{c_2})^0 c_{1i}\Big]$$

$$= 0$$

$$I_1^{V_2^2,1} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, (c_1^2+c_2^2-2c_1\cdot c_2)\, e^{-c_1^2} e^{-c_2^2}\, V_T^2 c_2^2 \left(c_1^2 - \frac{d+2}{2}\right) c_{1i} = 0$$

$$I_2^{1,V_1^2} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, (c_1^2+c_2^2-2c_1\cdot c_2)\, e^{-c_1^2} e^{-c_2^2}\, V_T^2 c_1^2 \left(c_{1i}c_{1j} - \frac{1}{d} c_1^2 \delta_{ij}\right)$$

$$= V_T^2 \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2-2c_1\cdot c_2)\left(c_1^2 c_{1i}c_{1j} - \frac{1}{d} c_1^4 \delta_{ij}\right)$$

$$= V_T^2 \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\Big[c_1^4 c_{1i}c_{1j} - \frac{1}{d} c_1^6 \delta_{ij} + c_1^2 c_2^2 c_{1i} c_{1j} - \frac{1}{d} c_1^4 c_2^2 \delta_{ij} - 2 c_1^2 (c_1\cdot\cancel{c_2})^0 c_{1i}c_{1j} + \frac{2}{d} c_1^4 (c_1\cdot\cancel{c_2})^0 \delta_{ij}\Big]$$

$$= V_T^2 \pi^d \left[\frac{d+4}{2d} \frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)} \delta_{ij} - \frac{1}{d}\delta_{ij} \frac{\Gamma(\frac{d+6}{2})}{\Gamma(d/2)} + \frac{d+2}{2d}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} \delta_{ij} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} - \frac{1}{d} \frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\right]$$

$$= V_T^2 \pi^d \delta_{ij}\left[\cancel{\frac{d+4}{2d}\frac{d+2}{2}}\frac{\cancel{d}}{2} - \frac{1}{\cancel{d}}\cancel{\frac{d+4}{2}\frac{d+2}{2}}\frac{\cancel{d}}{2} + \cancel{\frac{d+2}{2d}\frac{d}{2}}\frac{d}{2} - \frac{1}{\cancel{d}}\cancel{\frac{d+2}{2}\frac{d}{2}}\frac{d}{2}\right]$$

$$= 0$$

$$I_2^{V_2^2,1} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, (c_1^2+c_2^2-2c_1\cdot c_2)\, e^{-c_1^2} e^{-c_2^2}\, V_T^2 c_2^2 \left(c_{1i}c_{1j} - \frac{1}{d} c_1^2 \delta_{ij}\right)$$

$$= V_T^2 \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2-2c_1\cdot c_2)\left(c_2^2 c_{1i}c_{1j} - \frac{1}{d} c_1^2 c_2^2 \delta_{ij}\right)$$

$$= V_T^2 \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\Big[c_1^2 c_2^2 c_{1i}c_{1j} - \frac{1}{d} c_1^4 c_2^2 \delta_{ij} + c_2^4 c_{1i}c_{1j} - \frac{1}{d} c_1^2 c_2^4 \delta_{ij} - 2(c_1\cdot\cancel{c_2})^0 c_2^2 c_{1i}c_{1j} + \frac{2}{d}(c_1\cdot\cancel{c_2})^0 c_1^2 c_2^2 \delta_{ij}\Big]$$

$$= V_T^2 \pi^d \delta_{ij}\left[\frac{d+2}{2d} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} - \frac{1}{d} \frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} + \frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)} \frac{d}{2d} - \frac{1}{d} \frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} \frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)}\right]$$

$$= V_T^2 \pi^d \delta_{ij}\left[\cancel{\frac{d+2}{2d}\frac{d}{2}}\frac{\cancel{d}}{2} - \frac{1}{\cancel{d}}\cancel{\frac{d+2}{2}\frac{d}{2}}\frac{d}{2} + \cancel{\frac{d+2}{2}\frac{d}{2}\frac{d}{2d}} - \frac{1}{\cancel{d}}\frac{\cancel{d}}{2}\cancel{\frac{d+2}{2}}\frac{d}{2}\right]$$

$$= 0$$

$$\Longrightarrow \boxed{\xi_T^{(1)} = 0}$$

$$\xi_{ui}^{(1)} = \frac{1}{nV_T}\,\omega[f^{(0)}, V_i f^{(1)}] + \frac{1}{nV_T}\,\omega[V_i f^{(0)}, f^{(1)}]$$

$$I_1^{1,V_{1i}} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2-2c_1\cdot c_2)\left(c_1^2 - \frac{d+2}{2}\right) c_{1i} V_T c_{1i}$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} (c_1^2+c_2^2-2c_1\cdot c_2)\left(c_1^4 - \frac{d+2}{2} c_1^2\right)$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\Big[c_1^4 c_1^2 - \frac{d+2}{2} c_1^2 c_1^2 + c_1^2 c_2^2 c_1^2 - \frac{d+2}{2} c_2^2 c_1^2 - 2(c_1\cdot\cancel{c_2})^0 c_1^2 c_1^2 + (d+2)(c_1\cdot\cancel{c_2})^0 c_1^2\Big]$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\Big[c_1^6 - \frac{d+2}{2} c_1^4 + c_1^4 c_2^2 - \frac{d+2}{2} c_1^2 c_2^2\Big]$$

$$= V_T \pi^d \left[\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2} - \frac{d+2}{2}\frac{d+2}{2}\frac{d}{2} + \cancel{\frac{d+2}{2}\frac{d}{2}}\frac{d}{2} - \cancel{\frac{d+2}{2}\frac{d}{2}}\frac{d}{2}\right]$$

$$= V_T \pi^d \frac{d+2}{2}\frac{d}{2}\frac{1}{2}\left(d+4-d-2\right)$$

$$= V_T \pi^d \frac{d+2}{2}\frac{d}{2}\frac{1}{2}(2)$$

$$= V_T \pi^d \frac{d+2}{2}\frac{d}{2} \quad \checkmark$$

$$I_1^{V_{2i},1} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2} \left(c_1^2 + c_2^2 - 2\, c_1\cdot c_2\right)\left(c_1^2 - \frac{d+2}{2}\right) c_{1i}\, V_T\, c_{2i}$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left[c_1^4 - \frac{d+2}{2} c_1^2 + c_1^2 c_2^2 - \frac{d+2}{2} c_2^2 - 2 c_1^2 (c_1\cdot c_2) + (d+2)(c_1\cdot c_2)\right](c_1\cdot c_2)$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left[-2 c_1^2 (c_1\cdot c_2)^2 + (d+2)(c_1\cdot c_2)^2\right]$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left[-\frac{2}{d} c_1^4 c_2^2 + \frac{d+2}{d} c_1^2 c_2^2\right]$$

$$= V_T \frac{1}{d}\left[-2 \frac{\Gamma(\frac{d+4}{2})}{\Gamma(d/2)}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)} + (d+2)\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\frac{\Gamma(\frac{d+2}{2})}{\Gamma(d/2)}\right]$$

$$= V_T \frac{1}{d}\left[-2\frac{d+2}{2}\frac{d}{2}\frac{d}{2} + (d+2)\frac{d}{2}\frac{d}{2}\right]$$

$$= 0 \qquad \checkmark$$

$$I_2^{1,V_{1i}} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left(c_1^2 + c_2^2 - 2 c_1\cdot c_2\right)\left(c_{1i} c_{1j} - \frac{1}{d} c_1^2 \delta_{ij}\right) V_T\, c_{1i}$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left(\cancel{c_1^2}^{0} + \cancel{c_2^2}^{0} - 2 (c_1\cdot \cancel{c_2})^{0}\right)\left(c_1^2 c_{1j} - \frac{1}{d} c_1^2 c_{1j}\right)$$

$$= 0$$

$$I_2^{V_{2i},1} = \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left(c_1^2 + c_2^2 - 2 c_1\cdot c_2\right)\left(c_{1i} c_{1j} - \frac{1}{d} c_1^2 \delta_{ij}\right) V_T\, c_{2i}$$

$$= V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left(\cancel{c_1^2}^{0} + \cancel{c_2^2}^{0} - 2 c_1\cdot c_2\right)\left(c_1\cdot c_2\; c_{1j} - \frac{1}{d} c_1^2 c_{2j}\right)$$

$$= -2 V_T \int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\left[(c_1\cdot c_2)^2 \cancel{c_{1j}}^{0} - \frac{1}{d} c_1^2 (\cancel{c_1}\cdot c_2)^{0} c_{2j}\right]$$

$$= 0$$

Conclusion:

$$\xi_{ui}^{(1)} = \frac{1}{nV_T}\,\omega\left[f^{(0)}, V_i f^{(1)}\right] + \frac{1}{nV_T}\,\omega\left[V_i f^{(0)}, f^{(1)}\right]$$

$$= -\phi \frac{\cancel{nV_T}}{\pi^d}\frac{d(d+2)}{8(d-1)}\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\frac{1}{\cancel{nV_T}}\left(\kappa^* \frac{1}{T}\nabla_i T + \mu^* \frac{1}{n}\nabla_i n\right)\left(I_1^{1,V_{1i}} + I_1^{V_{2i},1}\right) - \phi \frac{n}{\pi^d}\frac{d+2}{4}\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}}\, \eta^* \nabla_i u_j \Big(\underbrace{I_2^{1,V_{1i}} + I_2^{V_{2i},1}}_{=0}\Big)$$

$$= -\phi \frac{1}{\cancel{\pi^d}}\frac{d(d+2)}{8(d-1)}\frac{\Gamma(d/2)\sqrt{2}}{\pi^{(d-1)/2}} V_T \cancel{\pi^d}\frac{(d+2)}{2}\frac{d}{2}\left(\kappa^* \frac{1}{T}\nabla_i T + \mu^* \frac{1}{n}\nabla_i n\right)$$

$$= -V_T\left(\kappa^* \frac{1}{T}\nabla_i T + \mu^* \frac{1}{n}\nabla_i n\right)\frac{d^2(d+2)^2}{8\,(d-1)}\,\phi\,\frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}}$$

<u>Coefficients</u> : avec le choix $\eta^*(p=0)=1$ (équivalent à $\nu_\eta^*(p=0)=1$) il vient :

$$\phi = \phi_{Maxwell}\ 4/(d+2)(d+4) \quad ; \quad \phi_{Maxwell} = \frac{4}{\sqrt{2}}\frac{\pi^{(d-1)/2}}{\Gamma(d/2)} .$$

Ainsi :

$$\nu_\eta^* = p\,\frac{2(d+2)^2}{(d+2)(d+4)} + (1-p) = p\,2\,\frac{d+2}{d+4} + 1-p$$

$$\nu_\kappa^* = \nu_\mu^* = p\,\frac{(d+2)(d+3)\,4}{2\,(d+2)(d+4)} + (1-p)\,\cancel{\frac{d^3-8d^2-20d+24}{8d\,(d+2)(d+4)}}\overset{\frac{(d-1)(d+4)}{d}\ \frac{4}{(d+2)(d+4)}}{4} = p\,2\,\frac{d+3}{d+4} + (1-p)\,\overset{\frac{4(d-1)}{d(d+2)}}{\cancel{\frac{-24+20d+8d^2-d^3}{2d(d+2)(d+4)}}}$$

$$\xi_n^{(0)*} = \nu_0^{-1} n\sigma^{d-1}\phi\, v_T\, d = \cancel{\frac{n\sigma^{d-1}}{n k_B T}}\ \frac{d+2}{\cancel{8}_{2}}\ \cancel{\frac{\Gamma(d/2)}{\pi^{(d-1)/2}}}\ \cancel{\frac{\sqrt{m k_B T}}{\sigma^{d-1}}}\ \sqrt{\cancel{\frac{2T}{m}}}\ d\,\frac{\cancel{4}}{\cancel{\sqrt{2}}}\,\cancel{\frac{\pi^{(d-1)/2}}{\Gamma(d/2)}}\ \frac{4}{\cancel{(d+2)}(d+4)} = \frac{2d}{d+4}$$

$$\xi_\tau^{(0)*} = \xi_n^{(0)*}/d = \frac{2}{d+4}$$

$$\xi_a^* = \frac{d^2(d+2)\cancel{4}}{8\,(d-1)}\ \frac{4}{\cancel{\sqrt{2}}}\ \cancel{\frac{\pi^{(d-1)/2}}{\Gamma(d/2)}}\ \cancel{\frac{\Gamma(d/2)\sqrt{2}}{4\,\pi^{(d-1)/2}}}\ \frac{\cancel{4}}{\cancel{(d+2)}(d+4)} = \frac{d^2(d+2)}{2(d-1)(d+4)}$$

Observe : coeff. de transport : • sphères dures borné par VHP si choix ϕ t.q. $\eta^*(p=0)=1$

• si choix ϕ t.q. $\chi^*(p=0)=1$, alors χ^*, μ^* bornés par VHP (ok), mais pas η^*

→ quel est le choix physique de ϕ ?

Correction des résultats VHP pour passer de la référence sphères dures → VHP

Le but est de normaliser les coefficients de transport de sorte à ce que pour $p=0$ ils donnent l'unité. On doit y arriver indépendamment de ϕ (fréq. de collision).
Ce sont ces grandeurs qui sont importantes dans l'analyse de stabilité linéaire.
Une autre étude est de regarder la valeur des coeff. de transport normalisés par rapport aux sphères dures. Par contre, dans ce cas on ne pourra pas avoir tous les coeff. de transport VHP normalisés à l'unité pour $p \to 0$.
On a calculé :

$$\nu_\eta(p) = \nu_\eta^*(p)\, \nu_0^{SD}$$

$$\eta^*(p=0) = \frac{\nu_0^{VHP}}{\nu_\eta(p=0)} = \frac{p^{(0)}}{\eta_0^{VHP}\, \nu_\eta(p=0)}$$

Ainsi :

$$\nu_\eta(p) = \phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \left[p\, \frac{(d+2)^2}{2} + (1-p)\frac{(d+2)(d+4)}{4} \right]_{p=0} \nu_0^{SD}$$

$$\eta^*(p=0) = \frac{p^{(0)}}{\eta_0^{VHP}\, \phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \left[p\, \frac{(d+2)^2}{2} + (1-p)\frac{(d+2)(d+4)}{4} \right]_{p=0} \nu_0^{SD}}$$

$$= \frac{p^{(0)}}{\nu_0^{SD}\, \eta_0^{VHP}\, \phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4}}$$

$$= 1$$

$$\Longrightarrow \eta_0^{VHP} = \frac{p^{(0)}}{\nu_0^{SD}\, \phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4}} = \overbrace{\frac{p^{(0)}}{\nu_0^{SD}}}^{=\eta_0^{SD}} \frac{1}{\phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4}} = \frac{p^{(0)}}{\nu_0^{VHP}}$$

$$\Longrightarrow \boxed{\begin{aligned} \eta_0^{VHP} &= \eta_0^{SD} \frac{1}{\phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4}} \\ \nu_0^{VHP} &= \nu_0^{SD}\, \phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4} \end{aligned}}$$

Ainsi on vérifie bien la normalisation

$$\eta^*(p) = \frac{\eta}{\eta_0^{VHP}} = \frac{\phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4}}{\nu_\eta^* - \frac{1}{2} p\, \zeta_T^{(0)*}}$$

$$= \frac{\phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \frac{(d+2)(d+4)}{4}}{\phi \frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}} \left[p\, \frac{(d+2)^2}{2} + (1-p)\frac{(d+2)(d+4)}{4} \right] - p \frac{p^{(0)}}{\nu_0^{SD}} n\sigma^{d-1} \phi v_T}$$

$$= \frac{1}{p\, \frac{2(d+2)}{d+4} + (1-p) - p\, \frac{2}{d+4}}$$

$$= \frac{1}{p\, \frac{2(d+2)}{d+4} + (1-p) - p\, \zeta_T^{(0)*}}$$

De façon similaire pour κ_0 :

$$\nu_\kappa(p) = \nu_\kappa^*(p)\, \nu_0^{SD}$$

$$\boxed{\kappa_0^{VHP} = x\, \frac{d(d+2)\, k_B}{2(d-1)\, m}\, \eta_0^{VHP}}$$

$$\kappa^*(p=0) = \frac{\kappa(p=0)}{\kappa_0^{VHP}} = \frac{\nu_0^{VHP}}{\nu_\kappa} \frac{d-1}{d} \frac{1}{x} = 1$$

Ainsi :

$$X = \frac{\cancel{V_0^{SD}}\,\cancel{\phi}\,\cancel{\frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}}}\;\frac{(d+2)(d+4)}{4}}{\cancel{V_0^{SD}}\,\cancel{\phi}\,\cancel{\frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}}}\;\frac{(d-1)(d+3)+1}{d}}\;\frac{d-1}{d}$$

$$= \frac{(d+2)(d+4)}{\cancel{4}}\;\cancel{\frac{2\,\cancel{8d}}{-24+20d+8d^2-d^3}}\;\frac{d-1}{\cancel{d}}$$

$$\boxed{X = \cancel{\frac{2(d-1)(d+2)(d+4)}{-24+20d+8d^2-d^3}}} \qquad \frac{(d+2)(d+4)(d-1)}{4\left[(d-1)(d+3)+1\right]}$$

A nouveau, on vérifie la normalisation : les Eqs. pour K^* et μ^* doivent être divisées par X

$$K^* = \frac{1}{V_K^* - 2p\,\xi_T^{(0)*}}\left[\frac{1}{2}\,p\,\xi_n^{(0)*}\mu^* + \frac{d-1}{d}\right]\frac{1}{X}$$

$$\mu^* = \frac{2}{2V_\mu^* - 3p\,\xi_T^{(0)*} - 2p\,\xi_n^{(0)*}}\;p\,\xi_T^{(0)*}K^*\,\frac{1}{X}$$

$$V_K^* = V_\mu^* = \phi\,\frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}}\left[p\,\frac{(d+2)(d+3)}{2} + (1-p)\,\overset{\frac{(d-1)(d+3)+1}{d}}{\cancel{\frac{-24+20d+8d^2-d^3}{8d}}}\right] \qquad \phi = \phi_M\,\frac{4}{(d+2)(d+4)}$$

Les taux de déclin restent inchangés (cf. p. 10), car ils sont divisés par V_0^{VHP} uniquement.

Pour ce qui est du modèle de Maxwell, avec la même méthode on obtient

$$\eta_0^M = \eta_0^{SD}\,\frac{1}{\phi\,\frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}}}$$

$$V_0^M = V_0^{SD}\;\phi\,\frac{\Gamma(d/2)\sqrt{2}}{4\pi^{(d-1)/2}}$$

$$X = 1$$

Le modèle de Maxwell est un cas très particulier avec un choix de ϕ qui corrige les 2 normalisations à la fois, ou bien autrement dit la relation entre K_0 et η_0 est la même que pour les sphères dures. Et le ϕ qui réalise la chose est justement celui de Santos.

Modèle VHP : calcul du $\boxed{a_2}$

Equation de Boltzmann :

$$(\partial_t + \underline{v}\cdot\underline{\nabla}) f(\underline{r},\underline{v};t) = p J_a[f,f] + (1-p) J_c[f,f]$$

$$J_a[f,g] = -\sigma^{d-1} \frac{\varnothing}{S_d V_T} \int_{\mathbb{R}^d} d\underline{v}_2 \int d\hat{\sigma}\ |V_{12}|^2\ f(\underline{r},\underline{v}_2;t)\, g(\underline{r},\underline{v}_1;t)$$

$$J_c[f,g] = \sigma^{d-1} \frac{\varnothing}{S_d V_T} \int_{\mathbb{R}^d} d\underline{v}_2 \int d\hat{\sigma}\ |V_{12}|^2 (b^{-1}-1)\, g(\underline{r},\underline{v}_1;t)\, f(\underline{r},\underline{v}_2;t)$$

Calcul du a_2 : équation de Boltzmann avec moments :

$$p J_a[f,f] = -p\sigma^{d-1} \frac{\varnothing}{S_d V_T} \int d\sigma \int_{\mathbb{R}^d} dv_2\ \frac{n^2}{V_T^{d2}} |V_{12}|^2\ \tilde{f}(c_1)\tilde{f}(c_2)$$

$$= -p\sigma^{d-1} \frac{\varnothing}{S_d V_T} S_d \frac{n^2}{V_T^{d2}} V_T^d V_T^2 \int_{\mathbb{R}^d} dc_2\ c_{12}^2 \tilde{f}(c_1)\tilde{f}(c_2)$$

$$= -p\sigma^{d-1} \frac{\varnothing n^2}{V_T^{d-1}} \int_{\mathbb{R}^d} dc_2\ c_{12}^2 \tilde{f}(c_1)\tilde{f}(c_2)$$

$$(1-p) J_c[f,f] = (1-p)\sigma^{d-1} \frac{\varnothing}{S_d V_T} \frac{n^2}{V_T^{2d}} V_T^d V_T^2 \int_{\mathbb{R}^d} dc_2 \int d\hat{\sigma}\ |c_{12}|^2 (b^{-1}-1)\ \tilde{f}(c_1)\tilde{f}(c_2)$$

$$= (1-p) \frac{\varnothing n^2}{S_d V_T^{d-1}} \tilde{I}[\tilde{f},\tilde{f}] \quad , \quad \tilde{I}[\tilde{f},\tilde{f}] = \int_{\mathbb{R}^d} dc_2 \int d\hat{\sigma}\ |c_{12}|^2 (b^{-1}-1) \tilde{f}(c_1)\tilde{f}(c_2)$$

De façon similaire, on a :

$$\frac{dn}{dt} = -p\omega(t) n \quad ; \quad \omega(t) = n(t) V_T(t) \sigma^{d-1} \int_{\mathbb{R}^d} dc_1 \int_{\mathbb{R}^d} dc_2 \int d\hat{\sigma}\ |c_{12}|^2 \tilde{f}(c_1)\tilde{f}(c_2)\ \frac{\varnothing}{S_d}$$

$$= n(t) V_T(t) \sigma^{d-1} \varnothing \int_{\mathbb{R}^{2d}} dc_1 dc_2\ |c_{12}|^2 \tilde{f}(c_1)\tilde{f}(c_2)$$

$$\omega_0 = n_0 V_{T0} \sigma^{d-1} \varnothing \langle c_{12}^2 \rangle$$

Donc :

$$p\frac{1+\alpha_e}{2}\omega_0 \left[-\xi + \gamma\left(d + c_1\frac{d}{dc_1}\right)\right]\tilde{f}(c_1) = -p\sigma^{d-1}\varnothing \int_{\mathbb{R}^d} dc_2\ c_{12}^2 \tilde{f}(c_1)\tilde{f}(c_2) + (1-p)\sigma^{d-1}\varnothing\, \tilde{I}[\tilde{f},\tilde{f}]$$

$$\Longrightarrow \boxed{\langle c_{12}^2\rangle \left[1 + \frac{1-\alpha_e}{2}\left(d + c_1\frac{d}{dc_1}\right)\right]\tilde{f}(c_1) = \tilde{f}(c_1)\int_{\mathbb{R}^d} dc_2\ c_{12}^2 \tilde{f}(c_2) - \frac{1-p}{p}\frac{1}{S_d}\tilde{I}[\tilde{f},\tilde{f}]} \quad : \text{indép. de } \varnothing \quad !!! \quad (1)$$

$$\alpha_e = \frac{\int dc_1 \int dc_2 \int d\hat{\sigma}\ c_{12}^2\ c_1^2\ \tilde{f}(c_1)\tilde{f}(c_2)}{\left[\int dc\ c^2 f(c)\right] \int dc_1 \int dc_2 \int d\hat{\sigma}\ c_{12}^2\ \tilde{f}(c_1)\tilde{f}(c_2)} = \frac{\langle c_{12}^2 c_1^2\rangle}{\langle c_1^2\rangle\langle c_{12}^2\rangle}$$

Méthode de la limite :

$$\boxed{\langle c_{12}^2\rangle\left[1 + d\frac{1-\alpha_e}{2}\right]\tilde{f}(0) = \tilde{f}(0)\langle c_1^2\rangle - \frac{1-p}{p}\frac{1}{S_d}\lim_{c_1\to 0}\tilde{I}[\tilde{f},\tilde{f}]}$$

Intégrant (1) sur $\underline{c}_1$ avec poids c_1^k, en utilisant

$$\int_{\mathbb{R}^d} dc\ c^k\left(d + c\frac{d}{dc}\right)\tilde{f}(c) = -k\langle c^k\rangle$$

il vient :

$$\langle c_{12}^2\rangle\left[\langle c_1^k\rangle + \frac{1-\alpha_e}{2}\langle c_1^k\rangle k\right] = \langle c_{12}^2 c_1^k\rangle - \frac{1-p}{p}\overbrace{\underbrace{\int_{\mathbb{R}^d} dc_1\ c_1^k\ \tilde{I}[\tilde{f},\tilde{f}]}_{\underset{k=2}{\overset{!}{=}} 0}}^{:=\mu_k}$$

: conservation de l'énergie cinétique des chocs

$$\Longrightarrow \boxed{\alpha_e = 1 + \frac{2}{k}\left(\frac{\langle c_{12}^2 c_1^k\rangle}{\langle c_{12}^2\rangle\langle c_1^k\rangle} - 1\right) + \frac{1-p}{p}\frac{2}{k}\frac{\mu_k}{\langle c_{12}^2\rangle\langle c_1^k\rangle}}$$

On choisit $k=2$: alors $\mu_2 = 0$ et :

$$\boxed{\begin{aligned} \alpha_e &= \frac{\langle c_{12}^2 c_1^2\rangle}{\langle c_{12}^2\rangle\langle c_1^2\rangle} \\ \langle c_{12}^2\rangle\left[1 + d\,\frac{1-\alpha_e}{2}\right]\tilde{f}(0) &= \tilde{f}(0)\langle c_1^2\rangle - \frac{1-p_1}{p_{\not{d}}}\lim_{c_1\to 0}\tilde{I}[\tilde{f},\tilde{f}] \end{aligned}}$$

Il s'agit du système à résoudre :

$$\tilde{f}(c) = \mathcal{M}(c)\left[1 + a_2 S_2(c^2)\right] \quad ; \mathcal{M}(c) = \pi^{-d/2} e^{-c^2}$$

$$S_2(c^2) = \frac{1}{2}c^4 - \frac{d+2}{2}c^2 + \frac{d(d+2)}{8}$$

Calcul de la limite :

$$\lim_{c_1\to 0}\tilde{I}[\tilde{f},\tilde{f}] = \tilde{I}_e + \tilde{I}_g$$

$$\tilde{I}_e = -\lim_{c_1\to 0}\int_{\mathbb{R}^d} dc_2\int d\hat{\sigma}\; |c_{12}|^2\,\tilde{f}(c_1)\tilde{f}(c_2)$$

$$\tilde{I}_g = \lim_{c_1\to 0}\int_{\mathbb{R}^d} dc_2\int d\hat{\sigma}\; |c_{12}|^2\,\tilde{f}(c_1^{**})\tilde{f}(c_2^{**})$$

donc :

$$\tilde{I}_e = -S_d\,\tilde{f}(0)\langle c_2^2\rangle = -S_d\,\mathcal{M}(0)\left[1 + a_2 S_2(c^2=0)\right]\int_{\mathbb{R}^d} dc_2\,|c_{12}|^2\tilde{f}(c_2)$$

$$= -S_d\,\frac{1}{\pi^{d/2}}\left[1 + a_2\frac{d(d+2)}{8}\right]\langle c_2^2\rangle$$

avec :

$$\langle c_2^n\rangle = \int_{\mathbb{R}^d} dc\; c^n\,\frac{1}{\pi^{d/2}}e^{-c^2}\left[1 + a_2\left(\frac{1}{2}c^4 - \frac{d+2}{2}c^2 + \frac{d(d+2)}{8}\right)\right]$$

$$= \frac{1}{\pi^{d/2}}\int_{\mathbb{R}^d} dc\, e^{-c^2}\left[c^n + a_2\left(\frac{1}{2}c^{4+n} - \frac{d+2}{2}c^{2+n} + \frac{d(d+2)}{8}c^n\right)\right]$$

$$= \frac{\Gamma\left(\frac{d+n}{2}\right)}{\Gamma(d/2)} + a_2\left[\frac{1}{2}\,\frac{\Gamma\left(\frac{d+n+4}{2}\right)}{\Gamma(d/2)} - \frac{d+2}{2}\,\frac{\Gamma\left(\frac{d+n+2}{2}\right)}{\Gamma(d/2)} + \frac{d(d+2)}{8}\,\frac{\Gamma\left(\frac{d+n}{2}\right)}{\Gamma(d/2)}\right]$$

$$= \frac{\Gamma\left(\frac{d+n}{2}\right)}{\Gamma(d/2)}\left[1 + a_2\left(\frac{1}{2}\,\frac{d+n+2}{2}\,\frac{d+n}{2} - \frac{d+2}{2}\,\frac{d+n}{2} + \frac{d(d+2)}{8}\right)\right]$$

$$= \frac{\Gamma\left(\frac{d+n}{2}\right)}{\Gamma(d/2)}\left[1 + a_2\left(\frac{n(n-2)}{8}\right)\right]$$

Ainsi :

$$\tilde{I}_e = -S_d\,\frac{1}{\pi^{d/2}}\left[1 + a_2\frac{d(d+2)}{8}\right]\left[1 + \frac{n(n-2)}{8}a_2\right]\overbrace{\frac{\Gamma\left(\frac{d+n}{2}\right)}{\Gamma(d/2)}}^{=d/2} \quad ; S_d = 2\pi^{d/2}/\Gamma(d/2) \quad , n=2$$

$$\boxed{\tilde{I}_e = -S_d\,\frac{1}{\pi^{d/2}}\,\frac{d}{2}\left[1 + a_2\frac{d(d+2)}{8}\right]}$$

Terme de gain:

$$\tilde{I}_g = \int_{\mathbb{R}^d} dc_2 \int d\hat\sigma\; c_{12}^2\Big|_{c_1=0} \tilde f\left[(c_2\cdot\hat\sigma)\hat\sigma\right]\tilde f\left[c_2-(c_2\cdot\hat\sigma)\hat\sigma\right]$$

$$= \int d\hat\sigma \int_{\mathbb{R}^d} dc_2\; c_2^2\, \tilde f[c_x]\, \underbrace{\tilde f[\underline{c_2} - c_x \underline{\hat\sigma}]} \quad ; \quad \underline{c_2} = c_x\hat x + \underline{c_\perp} \quad ; \quad c_x = (c_2\cdot\hat\sigma) \in \mathbb{R}$$

$$= \tilde f\left(\sqrt{c_2^2+c_x^2-2c_x \underbrace{c_2\cdot\hat\sigma}_{=c_x}}\right) \qquad c_2^2 = c_x^2+c_\perp^2$$

$$= \tilde f\left(\sqrt{\cancel{c_x^2}+c_\perp^2+\cancel{c_x^2}-2\cancel{c_x^2}}\right)$$

$$= \tilde f(c_\perp)$$

$$= \int d\hat\sigma \int_{\mathbb{R}^d} dc_2\, (c_x^2+c_\perp^2)\,\tilde f[c_x]\,\tilde f[c_\perp]$$

$$= S_d \int_{\mathbb R} dc_x \int_{\mathbb R^{d-1}} dc_\perp \left(c_x^2\, \tilde f[c_x]\tilde f[c_\perp] + c_\perp^2\, \tilde f[c_x]\tilde f[c_\perp]\right)$$

$$= S_d \int_{\mathbb R} dc_x\, c_x^2\, \tilde f[c_x]\int_{\mathbb R^{d-1}} dc_\perp\, \tilde f[c_\perp] + S_d\int_{\mathbb R} dc_x\, \tilde f[c_x]\int_{\mathbb R^{d-1}} dc_\perp\, \tilde f[c_\perp]\, c_\perp^2$$

Avec:

$$I_{n,\tilde d} = \int_{\mathbb R^{\tilde d}} dc\; c^n\, \tilde f[c] = \int_{\mathbb R^{\tilde d}} dc\; c^n \frac{1}{\pi^{d/2}} e^{-c^2}\left[1+a_2\left(\frac12 c^4 - \frac{\tilde d+2}{2}c^2 + \frac{\tilde d(\tilde d+2)}{8}\right)\right]$$

$$= \frac{1}{\pi^{d/2}}\int_{\mathbb R^{\tilde d}} dc\; e^{-c^2}\left[c^n + a_2\frac12\left(c^{n+4} - (\tilde d+2)c^{n+2} + \frac{\tilde d(\tilde d+2)}{4}c^n\right)\right]$$

$$= \langle c^n\rangle \frac{1}{\pi^{(d-\tilde d)/2}}$$

$$= \frac{\Gamma\left(\frac{\tilde d+n}{2}\right)}{\Gamma(\tilde d/2)}\left[1+a_2\frac{n(n-2)}{8}\right]\frac{1}{\pi^{(d-\tilde d)/2}}$$

Ainsi:

$$\tilde I_g = S_d\, I_{2,1}\, I_{0,d-1} + S_d\, I_{0,1}\, I_{2,d-1}$$

$$= S_d \frac{\Gamma\left(\frac{1+2}{2}\right)}{\Gamma(1/2)}\left[1+a_2\frac{2(2-2)}{8}\right]\frac{\Gamma\left(\frac{d-1}{2}\right)}{\Gamma(d/2)}\left[1+a_2\frac{0(0-2)}{8}\right]\frac{1}{\pi^{d/2}}$$

$$+ S_d\frac{\Gamma\left(\frac{1+0}{2}\right)}{\Gamma(1/2)}\left[1+a_2\frac{0(0-2)}{8}\right]\frac{\Gamma\left(\frac{d-1+2}{2}\right)}{\Gamma(d-1/2)}\left[1+a_2\frac{2(2-2)}{8}\right]\frac{1}{\pi^{d/2}}$$

$$= S_d\frac{1}{2\pi^{d/2}} + S_d\frac{d-1}{2\pi^{d/2}}$$

$$\boxed{\tilde I_g = S_d\frac{d}{2}\frac{1}{\pi^{d/2}}}$$

Conclusion:

$$\tilde I = \lim_{c_1\to 0}\tilde I[\tilde f,\tilde f] = S_d\frac{d}{2}\frac{1}{\pi^{d/2}}\left[1-1-a_2\frac{d(d+2)}{8}\right] = -S_d\frac d2\frac{1}{\pi^{d/2}}a_2\frac{d(d+2)}{8}$$

$$\boxed{\tilde I = -S_d\frac{1}{\pi^{d/2}}a_2\frac{d^2(d+2)}{16}}$$

Et on constate que

$$\tilde{f}(0)\langle c_1^2\rangle = -\frac{1}{S_d}\,\tilde{I}_e$$

Ainsi :

$$\tilde{f}(0)\langle c_1^2\rangle = \frac{1}{\pi^{d/2}}\quad\frac{d}{2}\quad\left[1+a_2\frac{d(d+2)}{8}\right]$$

<u>Reste α_e</u> :

$$\langle c_1^2\rangle = \frac{d}{2}$$

$$\langle c_{12}^2\rangle = 2\frac{d}{2}\left[1+\frac{a_2}{16d}\Big(d(4+0)-2d(2+0)+0\Big)\right] = d$$

$$\langle c_{12}^2 c_1^2\rangle = \frac{1}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1 dc_2\ c_1^2(c_1^2+c_2^2-2\overset{0}{\cancel{c_1\cdot c_2}})e^{-c_1^2}e^{-c_2^2}\left[1+a_2\left\{\frac{1}{2}c_1^4+\frac{1}{2}c_2^4-\frac{d+2}{2}c_1^2-\frac{d+2}{2}c_2^2+\frac{d(d+2)}{4}\right\}\right]$$

$$= \frac{1}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}\Big[c_1^4+c_1^2c_2^2+a_2\Big\{\frac{1}{2}c_1^8+\frac{1}{2}c_1^4c_2^4-\frac{d+2}{2}c_1^6-\frac{d+2}{2}c_1^4c_2^2+\frac{d(d+2)}{4}c_1^4$$

$$+\frac{1}{2}c_1^6c_2^2+\frac{1}{2}c_1^2c_2^6-\frac{d+2}{2}c_1^4c_2^2-\frac{d+2}{2}c_1^2c_2^4+\frac{d(d+2)}{4}c_1^2c_2^2\Big\}\Big]$$

$$= \frac{1}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2}e^{-c_2^2}\Big[c_1^4+c_1^2c_2^2+a_2\Big\{\frac{1}{2}c_1^8+\frac{1}{2}c_1^4c_2^4-\frac{d+2}{2}c_1^6-2(d+2)c_1^4c_2^2+\frac{d(d+2)}{4}c_1^4+c_1^6c_2^2$$

$$+\frac{d(d+2)}{4}c_1^2c_2^2\Big\}\Big]$$

$$= \frac{d+2}{2}\frac{d}{2}+\frac{d}{2}\frac{d}{2}+a_2\Big\{\frac{1}{2}\frac{d+6}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}+\frac{1}{2}\frac{d+2}{2}\frac{d}{2}\frac{d+2}{2}\frac{d}{2}-\frac{d+2}{2}\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}$$

$$-2(d+2)\frac{d+2}{2}\frac{d}{2}\frac{d}{2}+\frac{d(d+2)}{4}\frac{d+2}{2}\frac{d}{2}+\frac{d+4}{2}\frac{d+2}{2}\frac{d}{2}\frac{d}{2}+\frac{d(d+2)}{4}\frac{d}{2}\frac{d}{2}\Big\}$$

$$= \frac{d(d+1)}{2}+a_2\frac{d}{4}(d+2)\qquad +O(a_2^2)$$

<u>Conclusion</u> :

$$\alpha_e = \frac{2}{d^2}\left[\frac{d(d+1)}{2}+a_2\frac{d(d+2)}{4}\right] = \frac{d+1}{d}-a_2\frac{d+2}{2d}$$

$$\cancel{\alpha}\left[1+d\frac{1-\alpha_e}{2}\right]\frac{1}{\cancel{\pi^{d/2}}}\left[1+a_2\frac{d(d+2)}{8}\right] = \frac{1}{\cancel{\pi^{d/2}}}\frac{\cancel{\alpha}}{2}\left[1+a_2\frac{d(d+2)}{8}\right]-\frac{1-p}{p}\frac{1}{\cancel{8}}\left(-\cancel{8}\right)\frac{1}{\cancel{\pi^{d/2}}}a_2\frac{d(d+2)}{16}$$

$$\Rightarrow\left[1+\frac{d}{2}-\frac{d}{2}\alpha_e\right]\left[1+a_2\frac{d(d+2)}{8}\right] = \frac{1}{2}\left[1+a_2\frac{d(d+2)}{8}\right]+\frac{1-p}{p}a_2\frac{d(d+2)}{16}$$

$$= \frac{1}{2}+a_2\frac{d(d+2)}{16}\left(1+\frac{1-p}{p}\right)$$

$$= \frac{1}{2}+a_2\frac{d(d+2)}{16}\frac{1}{p}$$

Choisit le même schéma de linéarisation que pour l'annihil. prob. gaz. sph. dures :

$$1+\frac{d}{2}-\frac{d}{2}\alpha_e = \left[\frac{1}{2}+a_2\frac{d(d+2)}{16}\frac{1}{p}\right]\left[1-a_2\frac{d(d+2)}{8}\right]$$

$$\simeq \frac{1}{2}-a_2\frac{d(d+2)}{16}+a_2\frac{d(d+2)}{16}\frac{1}{p}\qquad+O(a_2^2)$$

$$\Rightarrow\quad \alpha_e = \frac{2}{d}\left[1+\frac{d}{2}-\frac{1}{2}+a_2\frac{d(d+2)}{16}\left[1-\frac{1}{p}\right]\right]$$

$$\Rightarrow\quad \alpha_e = \frac{2}{d}\frac{d+1}{2}+\frac{d+2}{8}a_2\frac{p-1}{p}$$

$$\Rightarrow\quad \alpha_e = \frac{d+1}{d}-a_2\frac{d+2}{8}\frac{1-p}{p}$$

Avec la valeur de de :

$$\frac{\cancel{d+1}}{\cancel{d}} + a_2 \frac{d+2}{2d}(d^3+4d^2-2) = \frac{\cancel{d+1}}{\cancel{d}} - a_2 \frac{d+2}{8}\frac{1-p}{p}$$

$$\Rightarrow \boxed{a_2 = 0} \quad \triangle!$$

Autre choix de linéarisation :

$$\left[1+\frac{d}{2} - \frac{d}{2}\left(\frac{d+1}{d} + a_2 \frac{d+2}{2d}(d^3+4d^2-8)\right)\right]\left[1 + a_2 \frac{d(d+2)}{8}\right] = \frac{1}{2} + a_2 \frac{d(d+2)}{16}\frac{1}{p}$$

$$\Rightarrow \left[\underbrace{1+\frac{d}{2} - \frac{d+1}{2}}_{=\frac{d+2-d-1}{2} = \frac{1}{2}} - a_2 \frac{d+2}{4}(d^3+4d^2-8)\right]\left[1+a_2\frac{d(d+2)}{8}\right] = \frac{1}{2}+a_2 \frac{d(d+2)}{16}\frac{1}{p}$$

$$\Rightarrow \boxed{\underbrace{\left[\frac{1}{2} - a_2 \frac{d+2}{4}(d^3+4d^2-8)\right]}_{a}\underbrace{\left[1+a_2\frac{d(d+2)}{8}\right]}_{b} = \underbrace{\frac{1}{2}+a_2 \frac{d(d+2)}{16}\frac{1}{p}}_{c}}$$ Eq. générale à l'ordre 2.

Structure : $a \cdot b = c$

Solutions :

⊗ $a = c/b \Rightarrow \boxed{a_2 = 0}$

□ $a \cdot b = c \Rightarrow \left(\cancel{\frac{1}{2}} + a_2 \frac{d(d+2)}{16} + a_2 \frac{1}{16}(d^3+4d^2-8)\right) = \cancel{\frac{1}{2}} a_2 \frac{d(d+2)}{16}\frac{1}{p} \Rightarrow \boxed{a_2 = 0}$

⊠ $1 = c/(ab) \Rightarrow \left[\frac{1}{2} + a_2 \frac{d(d+2)}{16}\frac{1}{p}\right] \Big/ \left\{\frac{1}{2}\left(1 + a_2 \frac{1}{8}(d^3+4d^2-8)\right)\left(1+a_2\frac{d(d+2)}{8}\right)\right\} = 1$

$$\Rightarrow \left[\frac{1}{2}+a_2\frac{d(d+2)}{16}\frac{1}{p}\right] 2\left[1 - a_2\frac{d(d+2)}{8} - a_2 \frac{d^3+4d^2-8}{8}\right] = 1$$

$$\Rightarrow \cancel{\frac{1}{2}} - \frac{1}{2}a_2 \frac{1}{8}\left[d(d+2) + d^3+4d^2-8\right] + a_2 \frac{d(d+2)}{16}\frac{1}{p} = \cancel{\frac{1}{2}}$$

$$\Rightarrow \boxed{a_2 = 0}$$

⊠ $\frac{ab}{c} = 1 \Rightarrow \left[\frac{1}{2} + a_2 \frac{1}{16}(d^3+4d^2-8)\right]\left[1+a_2\frac{d(d+2)}{8}\right]\left[1 - a_2 \frac{d(d+2)}{8}\frac{1}{p}\right] 2 = 1$

$$\left[\frac{1}{2} + a_2\frac{d(d+2)}{16} + a_2\frac{d^3+4d^2-8}{16}\right]\left[1-a_2\frac{d(d+2)}{8}\frac{1}{p}\right] = \frac{1}{2}$$

$$\cancel{\frac{1}{2}} - a_2\frac{d(d+2)}{16}\frac{1}{p} + a_2 \frac{d^2+2d+d^3+4d^2-8}{16} = \cancel{\frac{1}{2}}$$

$$\Rightarrow \boxed{a_2 = 0}$$

$$\left[\frac{1}{2} + a_2 \frac{d^3+4d^2-8}{16}\right]\left[1 - \underbrace{a_2\frac{d(d+2)}{8}\frac{1}{p} + a_2\frac{d(d+2)}{8}}_{= d(d+2)\left(\frac{p-1}{p}\right)}\right] = \frac{1}{2}$$

$$\cancel{\frac{1}{2}} + a_2 \frac{1}{16}\left(-\frac{d(d+2)}{p} + d(d+2)\right) + a_2 \frac{d^3+4d^2-8}{16} = \cancel{\frac{1}{2}}$$

$$\Rightarrow \boxed{a_2 = 0}$$

⊗ $\frac{1}{a} = b/c \Rightarrow 2\left[1 - a_2 \frac{1}{8}(d^3+4d^2-8)\right] = \left[1+a_2\frac{d(d+2)}{8}\right] 2\left[1 - a_2\frac{d(d+2)}{16}\frac{1}{p}\right]$

$$\Rightarrow \cancel{1} - a_2 \frac{1}{8}(d^3+4d^2-8) = \cancel{1} - a_2\frac{d(d+2)}{16}\frac{1}{p} + a_2\frac{d(d+2)}{8}$$

$$\Rightarrow \boxed{a_2 = 0}$$

□ $\frac{1}{ab} = \frac{1}{c} \Rightarrow 2\left(1 - a_2\frac{d(d+2)}{16} - a_2\frac{d^3+4d^2-8}{16}\right) = 2\left(1 - a_2\frac{d(d+2)}{16}\frac{1}{p}\right) \Rightarrow \boxed{a_2 = 0}$

△ $b = c/a \Rightarrow 1 + a_2\frac{d(d+2)}{8} = \left[\frac{1}{2} + a_2\frac{d(d+2)}{16}\frac{1}{p}\right] 2\left[1 - a_2 \frac{1}{8}(d^3+4d^2-8)\right]$

$$\Rightarrow \frac{1}{2} + a_2\frac{d(d+2)}{8 \cdot 2} = \frac{1}{2} - a_2\frac{1}{16}(d^3+4d^2-8) + a_2\frac{d(d+2)}{16}\frac{1}{p}$$

$$\Rightarrow \boxed{a_2 = 0}$$

△ $\frac{1}{b} = \frac{a}{c} \Rightarrow 1 - a_2\frac{d(d+2)}{8} = \left[\frac{1}{2} + a_2\frac{1}{16}(d^3+4d^2-8)\right] 2\left[1 - a_2\frac{d(d+2)}{8}\frac{1}{p}\right]$

$$\Rightarrow \cancel{\frac{1}{2}} - a_2\frac{d(d+2)}{16} = \cancel{\frac{1}{2}} - a_2\frac{d(d+2)}{16}\frac{1}{p} + a_2\frac{1}{16}(d^3+4d^2-8)$$

$$\Rightarrow \boxed{a_2 = 0}$$

<u>Conclusion</u>: les 8 développements de Taylor à l'ordre linéaire donnent tous $a_2 = 0$!

Pour aller à l'ordre quadratique en a_2, il faut reprendre les calculs de $\tilde{I}_e$ et $\tilde{I}_g$. On a à l'ordre a_2^2 :

$$\tilde{I}_e = -S_d \frac{1}{\pi^{d/2}} \frac{d}{2}\left[1 + a_2 \frac{d(d+2)}{8}\right] \qquad \text{(inchangé)}$$

$$\tilde{I}_g = S_d \frac{d}{2} \frac{1}{\pi^{d/2}} \qquad \text{(inchangé)}$$

$$\tilde{f}(0)\langle c_1^2\rangle = \frac{1}{\pi^{d/2}} \frac{d}{2}\left[1 + a_2 \frac{d(d+2)}{8}\right] \qquad \text{(inchangé)} \quad ; \; \tilde{f}(0) = \frac{1}{\pi^{d/2}}\left[1 + a_2 \frac{d(d+2)}{8}\right]$$

$$\langle c_1^2\rangle = \frac{d}{2} \qquad \text{(inchangé)}$$

Par contre $\langle c_{12}^2\rangle$ et $\langle c_{12}^2 c_1^2\rangle$ changent :

$$\langle c_{12}^2 c_1^{\overset{n=0,2}{n}}\rangle = \frac{1}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\, c_1^n (c_1^2 + c_2^2)\Big[1 + a_2\{\text{idem}\} + a_2^2\left(\frac{1}{2}c_1^4 - \frac{d+2}{2}c_1^2 + \frac{d(d+2)}{8}\right)$$
$$\times\left(\frac{1}{2}c_2^4 - \frac{d+2}{2}c_2^2 + \frac{d(d+2)}{8}\right)\Big]$$

$$= \{\text{idem}\} + \frac{a_2^2}{\pi^d}\int_{\mathbb{R}^{2d}} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\, c_1^n (c_1^2 + c_2^2)\left(\frac{1}{2}c_1^4 - \frac{d+2}{2}c_1^2 + \frac{d(d+2)}{8}\right)\left(\frac{1}{2}c_2^4 - \frac{d+2}{2}c_2^2 + \frac{d(d+2)}{8}\right)$$

$$= \{\text{idem}\} + \frac{a_2^2}{\pi^d}\int_{\mathbb{R}^d} dc_1 dc_2\, e^{-c_1^2} e^{-c_2^2}\Big[\frac{1}{4}c_1^{6+n}c_2^4 + \frac{1}{4}c_1^{4+n}c_2^4 (-64-32d) + \frac{1}{64}c_1^{6+n}c_2^2(-32-16d) + \frac{1}{64}c_1^{2+n}c_2^6(-32-16d)$$
$$+ \frac{1}{64}c_1^{6+n}(8d+4d^2) + \frac{1}{64}c_1^{6+n}(8d+4d^2) + \frac{1}{64}c_1^n c_2^6 (8d+4d^2)$$
$$+ \frac{1}{64}c_1^{4+n}c_2^2(64+72d+20d^2) + \frac{1}{64}c_1^{2+n}c_2^4(64+72d+20d^2)$$
$$+ \frac{1}{64}c_1^{2+n}c_2^2(-32d-32d^2-8d^3) + \frac{1}{64}c_1^{4+n}(-16d-16d^2-4d^3)$$
$$+ \frac{1}{64}c_1^n c_2^4(-16d-16d^2-4d^3) + \frac{1}{64}c_1^{2+n}(4d^2+4d^3+d^4)$$
$$+ \frac{1}{64}c_1^n c_2^2(4d^2+4d^3+d^4)\Big]$$

$$= \{\text{idem}\} + a_2^2 c_n$$

où :

$$\left.\begin{aligned} c_0 &= 0 \implies \langle c_{12}^2\rangle = d \\ c_2 &= 0 \implies \langle c_{12}^2 c_1^2\rangle = \frac{d(d+1)}{2} + a_2\frac{d(d+2)}{4}\end{aligned}\right\} \Rightarrow \alpha_e = \frac{\langle c_{12}^2 c_1^2\rangle}{\langle c_{12}^2\rangle\langle c_1^2\rangle} = \frac{2}{d^2}\left[\frac{d(d+1)}{2} + a_2\frac{d(d+2)}{4}\right] = \frac{d+1}{d} + a_2\frac{d+2}{2d}$$

$\Rightarrow$ L'équation à l'ordre quadratique est exactement la même !

$$2\frac{d}{\cancel{2}}\left[1 + \frac{d}{2} - \frac{d}{2}\left\{\frac{d+1}{d} + a_2\frac{d+2}{2d}\right\}\right]\frac{1}{\cancel{\pi^{d/2}}}\left[1 + a_2\frac{d(d+2)}{8}\right] = \frac{1}{\cancel{\pi^{d/2}}}\frac{d}{\cancel{2}}\left[1 + a_2\frac{d(d+2)}{8}\right]$$
$$-\frac{1-p}{p}\frac{1}{\cancel{S_d}}\left(-\cancel{S_d}\frac{1}{\cancel{\pi^{d/2}}}\frac{d}{\cancel{2}}\left[1 + a_2\frac{d(d+2)}{8}\right] + \cancel{S_d}\frac{d}{\cancel{2}}\frac{1}{\cancel{\pi^{d/2}}}\right)$$

$$\Rightarrow \left[1 + \frac{d}{2} - \frac{d+1}{2} - a_2\frac{d+2}{4}\right]\left[1 + a_2\frac{d(d+2)}{8}\right] = \frac{1}{2} + \frac{a_2}{2}\frac{d(d+2)}{8} - \frac{1-p}{p}\left\{-\cancel{1} - a_2\frac{d(d+2)}{8} + \cancel{1}\right\}$$

$$\Rightarrow \left[\frac{d+2-d-1}{2} - a_2\frac{d+2}{4}\right]\left[1 + a_2\frac{d(d+2)}{8}\right] = \frac{1}{2} + \frac{a_2}{2}\frac{d(d+2)}{8} + \frac{1-p}{p}a_2\frac{d(d+2)}{8}$$

$$\Rightarrow \left[\frac{1}{2} - a_2\frac{d+2}{4}\right]\left[1 + a_2\frac{d(d+2)}{8}\right] = \frac{1}{2} + a_2\frac{d(d+2)}{16}\,\frac{\cancel{p}+1-\cancel{p}}{p}$$

$$\Rightarrow \boxed{\left[1 - a_2\frac{d+2}{2}\right]\left[1 + a_2\frac{d(d+2)}{8}\right] = 1 + a_2\frac{d(d+2)}{8}\frac{1}{p}}$$

Solution :

$$\left[1 - a_2\frac{d+2}{2}\right]\left[1 + a_2\frac{d(d+2)}{8}\right] = 1 + a_2\frac{d(d+2)}{8}\frac{1}{p}$$

$$\Rightarrow \cancel{1} + a_2\frac{d(d+2)}{8} - a_2\frac{d+2}{2} - a_2^2\frac{d(d+2)^2}{16} = \cancel{1} + a_2\frac{d(d+2)}{8}\frac{1}{p}$$

$$\Rightarrow a_2\left[\frac{d(d+2)}{8}-\frac{d+2}{2}-a_2\frac{d(d+2)^2}{16}-\frac{d(d+2)}{8}\frac{1}{p}\right]=0$$

$$\Rightarrow a_2\,\frac{1}{8}\left[d(d+2)-4(d+2)-d(d+2)\frac{1}{p}-a_2\frac{d(d+2)^2}{2}\right]=0$$

$$\Rightarrow a_2\,\frac{1}{8}\left[d(d+2)\frac{p-1}{p}-4(d+2)-a_2\frac{d(d+2)^2}{2}\right]=0$$

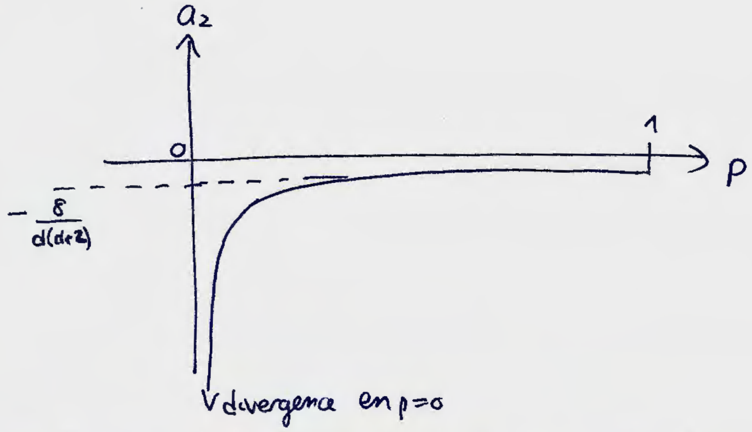
$$\Rightarrow a_2\,\frac{1}{8}\,\frac{d(d+2)^2}{2}\left[a_2+4(d+2)\frac{2}{d(d+2)^2}-d(d+2)\frac{p-1}{p}\frac{2}{d(d+2)^2}\right]=0$$

$$\Rightarrow a_2\,\frac{d(d+2)^2}{16}\left[a_2+\frac{8p}{pd(d+2)}-\frac{p-1}{p}\frac{2d}{d(d+2)}\right]=0$$

$$\Rightarrow a_2\,\frac{d(d+2)^2}{16}\left[a_2+\frac{8p-2dp+2d}{p\,d(d+2)}\right]=0$$

$$\Rightarrow a_2\,\frac{d(d+2)^2}{16}\left[a_2+2\,\frac{d+p(4-d)}{d(d+2)p}\right]=0$$

$$\Rightarrow \quad a_2=\begin{cases}0\\[4pt] -2\,\dfrac{d+p(4-d)}{d(d+2)p}\end{cases}$$



La solution a_2 non nulle n'est pas physique à cause de la divergence à l'origine. Conclusion pour l'état homogène : (tous dév. Taylor linéaires possibles, ordre non linéaire, solution ordre arbitraire)

$$\boxed{a_2(d,p)=0}$$

Etat homogène = maxwellienne !!!

Ceci va rendre la suite des calculs tout aussi simple que pour le modèle de Maxwell.